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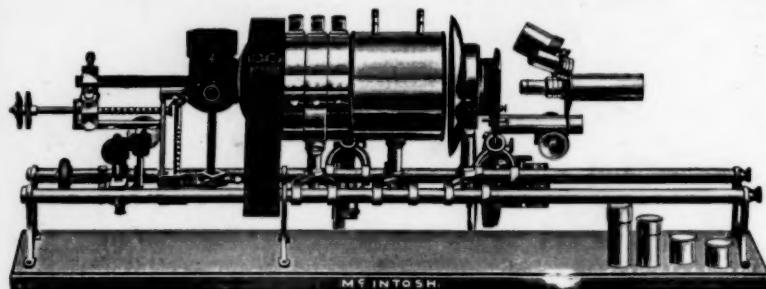
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WHOLE NO. 138

HOW ROUND IS THE SHAPE OF THE EARTH?

BY G. W. LITTLEHALES,

U. S. Hydrographic Office, Washington, D. C.

It is now nearly four hundred years since the rotundity of the earth was proved. Speculations had been formed and surmises had been uttered concerning the spherical form of the earth many generations before the memorable event which was to constitute the demonstration, and these had been alternately received and embraced, or rejected and spurned, by the learned, with the flow and ebb of civilization in the intervening centuries.

When the barbarians invaded and overran Europe during the fourth and fifth centuries, ancient society, as well as the sciences and the geographical knowledge of antiquity, was swept away. The advanced ideas of the Greek philosophers concerning the figure of the earth and the motions of the celestial bodies were forgotten, and were replaced by the crudest conceptions concerning natural things. An example will suffice to show how great was the retrogression that science suffered during the centuries of decadence. In the sixth century, Cosmas, who passed for a great geographer in his time, said that the doctrine of the antipodes was absurd, and that the earth was a quadrilateral plane, four hundred journeys, or stations of thirty miles each, in length, and two hundred in breadth. The degeneracy of geographical ideas is shown by the figures in the work of Cosmas in which he explains the form of the world by comparing it with the tabernacle of Moses. The stars were transported by angels, who were likewise charged with regulating the eclipses. The cause of the succession of day and night was referred to the interposition of a great mountain, behind which the sun dis-

appeared each evening. The firmament extended around the earth and the stars, enclosing them hermetically in its crystal walls.

The scientific views which animated the times of Strabo and Ptolemy were wholly absent. There was a disposition to twist facts so as to make them agree with what was believed to be religious truth, and this, continuing along the descending scale of the Dark Ages, led back eventually, through the suggestion of the Scriptural phrase, "the circle of the earth," to the Homeric idea that the earth had the appearance of a disc surrounded by the ocean.

Out of the lethargy of these centuries, by force of the necessities of commerce, the great ideas, so long exiled from the minds of men by adventurous speculations and by infantile conceptions concerning natural things, began at length to germinate again. The capture of Constantinople by the Turks in 1453 closed the overland trade routes to Asia, which, for many generations, had brought wealth to Venice and Genoa. Turkish pirates so overran the Eastern Mediterranean that Christian vessels were no longer safe. A powerful influence thus arose to transfer the center of civilization and commercial activity from the Mediterranean to the coasts of Western Europe; to turn attention to the Great Western Ocean for another route to India, Cathay, the islands of spices, and all the charms and riches of the East; to bring to the forefront the generation of Columbus and Gama, the generation which redoubled all that was previously known of the surface of the earth, the generation that gave America, the route to India, and, at its close, the achievement of Ferdinand Magellan.

Magellan, after a sojourn in the far East, returned home, and, having devoted himself to serious cosmographical studies, offered his services to the King of Spain with the purpose of reaching the Spice Islands by a new and shorter way than the route around Africa taken by the Portuguese under Vasco de Gama. He set sail on the 20th of September, 1519; in the following year, on the 21st of October, he entered the mouth of the passage in 52° south latitude, henceforth to be known as the Strait of Magellan. On the 28th of November, on leaving the Strait, he beheld the mighty ocean. For ninety-nine days, the vessels of Magellan ploughed the Pacific; on the 6th of March, 1521, the Mariana Islands rose before him, and ten days later the flotilla was in sight of the archipelago

which was to bear the name of the Philippines. Here Magellan lost his life in battle with the natives, but one of his vessels, the Victoria, in command of Sebastian del Cano, ultimately reached Spain, by way of the Indian Ocean, in 1522.

The voyage of Magellan was the greatest event in the most remarkable period of the world's history; it far surpassed all others in settling the fundamental conceptions of men concerning the earth which they inhabit. The discoveries of the preceding thirty years had resulted in adding the western continents to the chart of the world, in crossing the fiery zone of the ancients, in dealing a death blow to Ptolemy's view that the Indian Ocean was an enclosed sea, and in reaching the southern temperate zone of Aristotle and Mela. It was reserved to the expedition of Magellan to prove that the earth was round by going around it, and to show that the doctrine of the antipodes was no longer a scientific theory but a demonstrated fact. This event, by removing the insuperable obstacle to real knowledge which the prejudice of the ancients had established as a physical belief, and by telling a great truth in such a manner as to convince the understanding, is without parallel as a formative influence upon human thought; and its effect upon the minds of men is traceable throughout all those awakening changes which characterized the transformation of the age of mediævalism into the new intellectual life that was heralded by the achievements of Kepler, Galileo, Newton, and the Cassinis.

Men were no sooner satisfied that the earth was round than they made the precipitate supposition that its form was truly spherical, and they remained in this belief until the latter part of the seventeenth century when the French Academy sent the astronomer Richer to make observations of the sun's altitude at Cayenne, in equatorial South America. A remarkable circumstance which occurred in the course of this expedition was that the clock, though furnished with a pendulum of the same length as that which vibrated seconds at Paris, continued to lose nearly two and a half minutes a day at Cayenne. This created great astonishment in France, especially after the accuracy of it was confirmed by the observations of Varin and Deshayes, who, some years afterwards, visited different places in Africa and America, near the equator, and found the necessity of shortening the pendulum to make it beat seconds in those latitudes. The first

explanation of this remarkable phenomenon was given by Newton in the third book of his *Principia* published in 1687, where it is deduced as a necessary consequence of the earth's rotation on its axis and of the centrifugal force thence arising. The amount of this centrifugal force is greatest at the equator, and being measured by the momentary recess of any point from the tangent, which was known from the earth's rotation, it could be compared with the force of gravity at the same place, measured in like manner by the descent of a heavy body in the first moment of its fall. When Newton made this comparison, he found that the centrifugal force at the equator was the 289th part of the force of gravity, diminishing continually on going from thence toward the poles of the earth, where it ceases altogether. From the combination of this force, though small, with the force of gravity, it follows, that the direction in which bodies actually gravitate, or the direction of the plumb line, cannot tend exactly to the earth's center, and that the true horizontal line, such as is drawn by leveling, if continued from either pole in the plane of the meridian all around the earth, would not be a circle but an ellipse having its major axis in the plane of the equator and its minor axis coincident with the earth's axis of rotation. Inasmuch as the surface of the ocean itself actually traces this level as it extends from the equator to either pole, it was concluded that the teraqueous mass which we call the globe must be in the form of what geometers call an oblate spheroid, or a solid generated by the revolution of an elliptic meridian about its minor axis.

From the figure of the earth thus determined, Newton showed that the intensity of the force of gravity at any point on the surface is inversely as the square of the distance of that point from the center, and its decrease, therefore, in going from either pole to the equator is in the same ratio in which the degrees of the meridian decrease. It follows that a pendulum of given length would vibrate more slowly when carried from Europe into the torrid zone.

It was reserved for a more advanced condition of the theory of gravitation to give to the solution of the problem all the accuracy of which it is susceptible. It is a part, and a distinguishing part, of the glory of this discovery, that it was susceptible of more perfection than it received from the hands of the author; and that the centuries which have

elapsed since its inception have been continually adding to its perfection. This character belongs to a system which has truth and nature for its basis, and had not been exhibited in any of the physical theories that had before appeared in the world.

It is certain that no knowledge was transmitted to us from antiquity concerning the magnitude of the earth, save alone the mathematical principle upon which it was to be determined. Eratosthenes of Alexandria, who lived in the third century before the Christian era, attempted to measure an arc of the meridian in perfect conformity with this principle, but by means very inadequate to the importance and difficulty of the problem. He noticed that at Syene, in Southern Egypt, the sun at the summer solstice, being exactly in the zenith, cast no shadow of a vertical object, while at Alexandria, in Northern Egypt, the rays of the sun at the same time of the year made an angle with the vertical of one-fiftieth part of four right angles. From this, he concluded that, supposing the earth to be of spherical form, its circumference would be fifty times the distance between these two places. With a lack of attention to the methods of obtaining accurate data for the application of mathematical principles, which was characteristic of the infant state of the art of experiment and observation in that age, Eratosthenes not only failed to ascertain that Alexandria and Syene were not quite due north and south of each other, but he also neglected to find their distance apart with higher accuracy than the estimates of travelers would afford.

Such in its simplest form is the conception of the geodetic operations usually called the measurement of the arc of a meridian, which for its successful execution demands the most accurate instruments, the best observers, and long-continued labor. It is simply to measure the distance between two points on the same meridian of the earth, and find their difference of latitude. During the generations in which no doubt was entertained of the truly spherical form of the earth, and consequently of the equality of all the degrees of the meridian, repeated attempts were made by the most advanced geometers to measure arcs of the meridian with greater and greater care, under the belief that one accurately measured arc would be sufficient. But, since it is usually impracticable to find a line of sufficient length running due north and south

and level enough to be directly measured with the implements for linear measurement, the determination of the length of the observed line of the meridian continued to be attended with great difficulty and uncertainty until William Snell, a Dutch mathematician, put into practice, in the measurement of an arc in Holland, the methods of triangulation which have since been made the basis of all trigonometrical surveying. By this method, a long chain of triangles is formed between the places whose distance apart is to be found. The sides of these triangles are the lines which connect stations that have been laid out in the intervening region between the terminal places. One, at least, of the sides is located on a level plain where it may be precisely measured by special implements; and all the angles of the triangles being carefully observed, the lengths of the sides may be calculated, and hence the distance between the parallels of latitude passing through the terminal points. Snell gave an account of his operations in 1617, but, because of certain errors which remained to be corrected by him, no advantage was derived by the world from his work until its value was lost in other determinations of a more precise nature that were made in France after the adaptation of the telescope to circular instruments for measuring the angles of the triangulation.

Of course, the facts of nature concerning the earth were the same before Newton lived as they were afterwards, but the effect of his discoveries was to raise mankind to a higher level from which, as from a height ascended, a clearer view of natural conditions was obtained, and when the flattening of the earth at the poles was at length confirmed by the measurements that were undertaken in the equatorial and the arctic regions to set at rest the dispute among the philosophers of that day, it became evident that the determination of the size and shape of the earth could no longer be rested upon one accurately measured meridian arc.

Nothing is so hostile to the interests of truth as a fact inaccurately observed. Of this there is a remarkable example in the results given out in 1720 of the measurement of an arc of the meridian across France, from Amiens to Perpignan, although it was executed by Cassini, one of the first astronomers of Europe. According to that measurement, the degrees seemed to diminish on going from south to north, each being less by about one six-hundredth part than that which

immediately preceded it to the southward. From this result, which is entirely erroneous, the conclusion first deduced was correct, the error in reasoning, by a singular coincidence, having obscured the real meaning of the measurements upon which it was founded. Fontenelle argued that, as the degrees diminished in length in going toward the poles, the meridian must be less than the circumference of the equator, and the earth, of course, swelled out in the region of that circle, agreeably to the facts that had been observed concerning the retardation of the pendulum when carried to the south. This, however, was the direct contrary to the conclusion which ought to have been drawn, as was soon perceived by Cassini and by Fontenelle himself. The degrees growing less as they approached the pole was an indication of the curvature growing greater, or of the longer axis of the meridian being the line that passed through the poles and that coincided with the axis of the earth. The figure of the earth would, therefore, be that of a prolate spheroid, or one extended at the poles, such as would be formed by the revolution of an ellipse about its longer axis.

All this seemed quite inconsistent with the observations on the pendulum, as well as with the conclusions which Newton had deduced from the theory of gravitation. The French Academy of Sciences was thus greatly perplexed, and uncertain to what side to incline. In these circumstances, Cassini, whose errors were the cause of all the controversy, had the merit of suggesting that the only means by which the question concerning the figure of the earth was likely to receive a satisfactory solution was through the measurement of two meridian arcs, the one near the equator and the other as near the pole as the nature of the undertaking would admit.

So the famous measures by the French academicians of the Peruvian and Lapland arcs came to be made in the early part of the eighteenth century, with such results that the increase of the degrees toward the pole, or the oblateness of the earth's figure, was completely ascertained. Cassini, on resuming his own operations, discovered and candidly acknowledged the errors of his first measurement, and thus the objections which had arisen in this quarter against the theory of gravity became irresistible arguments in its favor, and marked the establishment of the Newtonian philosophy on the continent of Europe. And when the measured meridian

arcs in Lapland, Peru, and France were taken together to form a basis for finding the shape and magnitude of the earth, it was as if three wires, bent to the respective forms of these arcs on a certain scale, had been handed to an artisan skilled in the fashioning of models, with instructions to produce a spheroid of revolution to which the bent wires, when placed in their proper latitudes, would conform more closely than they would to any other spheroid. Many additional wires, bent to the exact forms of arcs in many parts of the world, and sometimes, as in the case of the United States, templates of large areas, are passed into the hands of the fashioning artisan through the great expansion of trigonometrical surveying in all parts of the world during the last century, and with the lapse of time, model spheroids are produced more and more closely representing the form and position which the mean sea level would have if the ocean extended under the continents and over all the globe.

This, in effect, is the nature of the result that has been reached through the mathematical treatment of the observations that have attended the measurement of the triangulations, only the mathematicians have long since reached a precision, in assigning the dimensions of the terrestrial spheroid, which is beyond the power of the artisan to calibrate.

The result has not been intended to pronounce that the actual form of the earth is an oblate spheroid whose major and minor semi-axes are 20,926,083 and 20,855,825 English feet; but that a spheroid of these dimensions conforms more closely to the surface of the earth than any other spheroid will. Whatever the real figure of the earth may be, if in the investigation it is supposed to be a spheroid, the arithmetical process can do no better than to bring it out a spheroid. Knowledge in this respect, as derived from arc measures, is deceptive in proportion as we lose sight of the significance of the fundamental assumption that the figure is a regular one. The forms of the regions of the earth's surface whose curvatures are measurements determine are local and particular, and, when the measurements are combined in the mathematical solution, the local characteristics are lost in the derived mathematical form or spheroid, whose surface consequently passes in and out, by small but unknown amounts, now above and now below the actual surface of the ocean, as disturbed by the attraction of the islands and continents, and the actual surface

of the land, as diversified by valleys and mountain masses.

The earth is, in fact, earth-shaped. Not conforming in homogeneity and regularity with the long-prevalent doctrine that it must have been formerly a liquid or molten earth in order to have acquired its flattened form at the poles as a result of rotation, but exhibiting wide variations in density and direction of the gravitational force, and responding in a continuous tremor of minute rhythmic and elastic movements of the surface to the action of the tide-producing bodies, and yielding by fracture and quaking to the accumulating stresses produced by the weight of the continents and the age-long shifting of masses of matter from region to region.

And what features shall be looked for with the unrolling of the centuries to come? Aside from the alterations which the works of time bring about in the solid crust of the earth, there are indications that the form of the surface waters alters from age to age in a degree so much greater as to change the whole face of nature.

The paucity of geodetic measurements in the southern hemisphere has thus far precluded a demonstration of the belief that there is a lack of symmetry between the northern and the southern hemispheres, but there is reason for concluding that this is the case, and that in the present age the size of the southern hemisphere is greater than that of the northern.

The earth moves each year throughout an elliptical orbit at one of whose foci is situated the sun, and rotates each day about an axis which is inclined nearly $66\frac{1}{2}^{\circ}$ to the plane of that orbit. If the earth were perfectly spherical in form, although its axis might be inclined, it would forever continue at a given season to arrive at the same point in its orbital path, and the same would take place even with the spheroidal form which does actually characterize it, if its axis were at right angles to the plane of the elliptical path. But as the axis is inclined and as the form is not spherical, the effect of the attraction of the sun and moon upon the bulging portion of the earth in the vicinity of the equator is to cause a progressive change in its direction, with a resulting retrograde motion of the line joining the center of the sun and the center of the earth at the points of the orbit where the axis of the earth stands perpendicular to this line. These points are called the vernal and autumnal equinoxes, and their retrograde movement is called the precession of the equinoxes.

At points in the earth's orbit equally removed from the equinoxes are the summer and winter solstices. For many centuries, the earth's orbit has been so situated in the ecliptic plane that the winter solstice of the northern hemisphere has nearly coincided with the point of nearest approach to the sun, called perihelion, and it thus arises that the half of the year corresponding to winter is about seven days longer in the southern hemisphere than in the northern, and that during the year the south pole has about 170 hours more of night than of day, while the north pole has an equal preponderance of light over darkness. From these reasons, and the added one that the winter of the northern hemisphere takes place when the earth is nearest to the sun and the winter of the southern hemisphere when the earth is farthest from the sun, it appears that in the present age the southern hemisphere is annually receiving less heat than the northern, with the consequent accumulation over the antarctic continent of a great mass of ice whose attraction is sufficient to drag the waters toward it until the level of the surrounding ocean is raised many feet above what would otherwise be its main position.

Somewhat less than seven centuries ago, the agencies that have produced this effect, by which the terrestrial spheroid is rendered ovaloidal or egg-like in shape, where exercising their greatest influence, for then the northern winter solstice came exactly at perihelion and the deficiency of annual heat in the southern hemisphere was at its maximum. With the slow precession of the equinoxes in the intervening centuries, the tendency must have been toward an arrest or lessening of the accumulation of ice over Antarctica, a gradual recession of the ocean from the far-southern lands, and perhaps a scarcely perceptible deepening of the harbors in the lower latitudes of the northern hemisphere; and this tendency of the recession of the ocean from the southern hemisphere and its gradual transference to the northern hemisphere must continue from century to century until, having passed through the age in which symmetry between the two hemispheres will prevail, there will come at length a stage, in the vast changes which will then have been effected in the geography of the world, when the earth will be nearest to the sun at the time of the southern winter solstice, and those conditions will be reversed which prevailed in their highest intensity in the southern hemisphere when the world's seats of civilization still clustered around the shores of the Mediterranean Sea.

LABORATORY USES OF THERMOS BOTTLES.¹

BY RAYMOND B. ABBOTT,
Berkeley, Calif.

The thermos bottle may be used to great advantage as a calorimeter in a large number of experiments in the laboratory. The losses due to radiation and conduction are very small when the temperature of the contents of the bottle is not too far above or below that of the surrounding medium. Even with a thermos bottle, it is necessary to keep the contents at a temperature as close as possible to that of the surrounding medium, to avoid appreciable errors. The value of results obtained will depend entirely upon the accuracy with which the thermal capacity of the bottle and contents is determined.

The water equivalent or thermal capacity may be measured very accurately, if care is taken to get the temperatures of the contents and all inner parts of the bottle well equalized. This may seem simple at first, but in reality it takes care to get consistent results.

If a mercury-glass thermometer is used, it must not be raised or lowered, but remain fixed in one position throughout the experiment. The thermometer must be and remain in the same position as above when the thermal capacity of the bottle is determined.

In order to get all the inner parts and contents of the bottle at equal temperatures, it is absolutely necessary to turn it upside down several times. A rubber washer at the neck will prevent liquids from leaking out.

If it is necessary to put a liquid into the bottle without a change of temperature, it must be brought very nearly to room temperature before pouring in. To get accurate weights of liquids, weigh them after they are poured into the bottle and subtract the weight of the bottle.

If the above precautions are taken into consideration during a series of observations, very consistent results may be obtained. Otherwise, all the advantages of the thermos bottle over the ordinary laboratory calorimeter may be lost.

Let me explain the case of a pint thermos bottle in which all the precautions were taken into consideration, with the exception of turning it upside down. The water equivalent was

¹Presented at a meeting of the Pacific Coast Physics Teachers, December 18, 1915, University of California, Berkeley, Cal.

sought for by two methods, as follows: 1. Method of mixtures.
2. Method of electrical heating.

The first method is known to all. The second method made use of a heating coil of resistance 1.037 ohms, an ammeter, which reads to $1/100$ amperes, a thermometer which reads to $1/100$ degrees Centigrade, and a stop watch which reads to one-fifth of a second. Distilled water was used in the bottle. Mechanical equivalent of heat used was 4.187 joules per calorie.

METHOD.

First.	Second.
16.9	18.3
15.3	13.7
17.0	21.0
23.5	25.6
14.9	17.2
13.5	16.9
21.7	19.8
21.3	20.5
13.4	19.7
Average 17.5	19.5
	21.3
	24.8
	24.3
	27.3
	24.5
	22.4
	20.0
	19.5
	21.0
Average	20.8
Average of 1 and 2 = 19.1	

The results show the same range of values by both methods, but nothing consistent about them by either.

By observing all the precautions named above, the following values were obtained:

Water Equivalent.

1	2
19.2	18.7
17.2	19.5
18.6	18.1
19.0	17.2
18.4	17.3
18.7	17.2
	17.0
	17.2
	18.0
Average 18.4	Average 17.7

Average of 1 and 2 = 18.05.
Grand total average, 18.5.

With the water equivalent of the bottle so accurately measured, it is possible to use the bottle for a number of purposes in the laboratory.

The calorimeter measures energy, and, therefore, if the resistance of the heating coil is accurately determined, we may use the thermos bottle to measure power, current, voltage, and even magnetic field strength as well as specific heat, latent heat, etc. It is quite an easy matter to check Rowland's value of the mechanical equivalent, 4.187 within two- or three-tenths of one per cent with the above apparatus.

TABULATED DATA AND RESULTS.

J = Mechanical equivalent of heat, 4.187 joules per cal.

H = Calories of heat.

m = Mass of water in grams.

T = Time in seconds.

R = Resistance in ohms, 1.037 ohms.

I = Current in amperes.

Q = Quantity of electricity.

t = Temperature change degrees Centigrade.

c = Water equivalent in gram calories per degree.

W = Power in watts.

Law of heating: $JH = I^2RT$.

$$H = (m + c) t.$$

$$J = \frac{I^2RT}{H}.$$

$$I = \sqrt{JH/RT}.$$

$$R = \frac{JH}{I^2T}.$$

$$W = I^2R = \frac{JH}{T}.$$

$$E = IR = W/I = \frac{JH}{IT} = \frac{\sqrt{JHR}}{T}$$

$$Q = IT = T\sqrt{JH/RT} = \sqrt{JHT/R}.$$

By using more water than above, and allowing for the change in the specific heat of water at different temperatures, errors could easily be reduced to two- or three-tenths of one per cent.

The resistance of the coil can be found in this way about as accurately as with a Wheatstone's bridge. The current and voltage are determined from reading the instrument scales.

If a tangent galvanometer be placed in the circuit so that the current and deflection are known, the reduction factor may be calculated accurately.

$$I = K \tan \theta,$$

$$K = 1/\tan \theta = \frac{V(JH/RT)}{\tan \theta},$$

$$\text{also } K = \frac{10F}{2\pi n/r} = 10F/G.$$

F = Horizontal component of the earth's magnetic field.

n = Number of turns of wire.

R = Radius of turns of wire.

$$F = KG/10 = \frac{2\pi n \sqrt{(JH/RT)}}{10r \tan \theta}.$$

The value of G can be calculated from the number of turns used on the galvanometer and their radius.

We see then that from the value of the current determined by the calorimeter, it is possible to find the value of F accurately.

It seems possible, from the above considerations, to use the thermos bottle together with a heating coil whose resistance is known, for purposes in the laboratory as follows: To determine specific heats and latent heats, to calibrate ammeters, voltmeters and wattmeters, to find the reduction factor of a tangent galvanometer and the earth's magnetic field.

Data Including J and R.						Calorimeter Results.				Correct Values.			
M	T	t	C	J	R	I	W	E	Q	I	W	E	Q
150	420	5.60	18.4	4.187	1.037	3.01	9.43	3.12	1267	3.00	9.33	2.11	1260
150	480	6.35	18.4	4.187	1.037	3.00	9.36	3.11	1442	2.99	9.33	3.11	1435
150	420	5.60	18.4	4.187	1.037	3.01	9.43	3.12	.267	3.00	9.33	3.11	1260
150	840	11.15	18.4	4.187	1.037	3.00	9.38	3.11	2522	3.00	9.33	3.11	2520

Data Including J and I.						Calorimeter Results.				Correct Values.			
M	T	t	C	J	I	R				R			
150	420	5.60	18.4	4.187	3.00		1.042				1.037		
150	480	6.35	18.4	4.187	2.99		1.045				1.037		
150	420	5.60	18.4	4.187	3.00		1.042				1.037		
150	840	11.15	18.4	4.187	3.00		1.039				1.037		

Data Including I and R.						Calorimeter Results.				Correct Values.			
M	T	t	C	I	R	J				J			
150	420	5.60	18.4	3.00	1.037	4.155				4.187			
150	480	6.35	18.4	2.99	1.037	4.160				4.187			
150	420	5.60	18.4	3.00	1.037	4.155				4.187			
150	840	11.15	18.4	3.00	1.037	4.180				4.187			

Later, a student performed this experiment in the laboratory, using a thermos bottle as a calorimeter. Data and results are as follows:

Data Including I and R.						Calorimeter Results.				Correct Values.			
M	T	t	C	I	R	J				J			
288.70	480	6.16	17.5	4.00	1.025	4.1734				4.187			
259.14	480	6.80	17.5	4.00	1.025	4.1906				4.187			
248.25	480	7.13	17.5	4.00	1.025	4.1654				4.187			
249.07	480	7.11	17.5	4.00	1.025	4.1643				4.187			

Average—4.1734.

Per cent error—0.32 per cent.

Sgd.: H. D. Draper.

PHYSICS LABORATORY PRACTICE AMELIORATION.

BY RALPH C. HARTSOUGH, M. A.,

Tsing Hua College, Peking, China.

The stimulation of original thought should be the chief end of all laboratory work. It is the author's firm conviction that laboratory investigation is the vital part of all sciences, and that it should be provocative of thought, developing the initiative in the student. This is where approximately ninety per cent of all laboratory exercises fail. Most of the experiments are like following a recipe in cooking a new food, only without the same interest in the end. We give the student apparatus, full detailed printed instructions, blanks for data, and the chief interest of the student is to get the data as soon as possible, receive credit for that experiment, and repeat the process with another.

Unless the teacher has the time to take up each individual experiment with the student when it is completed, there is no way of clinching one fact that the experiment may aim to teach. In most high schools, time does not permit the teacher to completely quiz each student about his or her experiment.

Presented with the above facts and the evils of copying, losing time, and superficial mechanical work of the students in the physical laboratory (perhaps the same conditions prevail in other laboratories) caused the author to study conditions with a purpose to remedy at least some of the most glaring faults.

We have endeavored to correct these conditions as outlined above, and do not pretend to have a perfect method. The results of much testing with various classes of students have proven that his throwing of the student more upon his own powers is productive of interest, initiative, and resourcefulness. By the ordinary method of conducting laboratory work, the author found it impossible to keep the student from being mechanical in the use of the most approved manuals.

The method used here is "your own method." Only references are given, an outline and some questions or illustrative problems. Of course, this system necessitates a reference book shelf in the laboratory (a necessity in any laboratory).

To stimulate initiative and interest, the author has practiced (1) variety of experiments to suit the varied interests of the students, (2) allow students some latitude in the choice of experiments, (3) make experiments problematic rather than a taking of data.

Before the student can get apparatus and start the experiment, he must outline to the instructor his method of attacking the experiment assigned. This checks any useless work and also prevents one student leaning on another's work.

The following is a sample of the experimental page placed in the hands of the student.

Name Date begun

Number Date finished

I. Definitions.

- (a) Resistance.
- (b) Kilowatt hour.

II. Object of experiment.

- (a) To find the cost per hour of operating an electric iron.

III. State your method briefly.

IV. Apparatus needed.

V. Tabulate your data, also sketch apparatus.

VI. Give sources of error.

VII. Questions.

- (a) Devise a scheme whereby a 110-volt iron could be used on a 220-volt circuit.

- (b) Is the resistance of the iron more when hot than cold? How much?

The author has found the following results, which are worthy of mention.

SUMMARY.

The student, by having thoroughly mastered the why and how of an experiment, accomplishes the rest with confidence and interest.

At the start, with this system, students will not get over as many experiments in a given length of time, but they will have them better, and after a few months will work just as rapidly as under the ordinary method.

The instructor's check before the student receives apparatus saves time in useless work for the students, also serves to check copying and one student leaning on another's work.

It is impossible for a student to start on an experiment and have a very hazy idea of what the different pieces of apparatus are. This formerly was found very often to be the case.

The author will be glad to answer any questions about points brought out or any other which may come to the attention of the reader.

OUTPUT OF ABRASIVES.

According to the annual statement of the Geological Survey on abrasive metals in 1915, now available for distribution, the value of natural abrasives produced and marketed in the United States during the year reached \$1,662,055.

AN EXPERIMENT IN PHOTOSYNTHESIS.

BY JOHN L. DAHL,

Normal School, Fredonia, N. Y.

The following very simple experiment does not, I believe, occur in the form submitted in any of the popular texts. It will doubtless prove of value to teachers of botany because of its simplicity and concrete application to photosynthesis.

This experiment may follow that proving the necessity of sunlight for starch-making, or it may be used for the twofold purpose of proving the need for sunlight and carbon dioxide in photosynthesis.

OBJECT: What results when green leaves are fed carbon dioxide and are subjected to direct sunlight?

APPARATUS: Two battery jars (or four), each with a glass funnel, ring stands, carbon dioxide generator, and a quantity of fresh water cress.

PROCEDURE: Place water cress plants with much leaf exposure under two funnels, and submerge each in a battery jar filled with *clear* water at the temperature of the room. Charge one jar with abundance of carbon dioxide by allowing the gas to bubble through the water and the plants. Place a test tube filled with water over the end of each funnel in order to collect whatever gas is generated by the plants. Place both jars, side by side, in direct sunlight. As a control, two jars similarly treated may be placed away from the direct sunlight. Thus:

Jar No. 1 charged with carbon dioxide and in direct sunlight.

Jar No. 2 not charged with CO_2 , and in direct sunlight.

Jar No. 3 charged with CO_2 , away from direct sunlight.

Jar No. 4 not charged with CO_2 , away from direct sunlight.

RESULTS: One cannot fail to collect at least three test tubes of oxygen in as many hours in the jar charged with carbon dioxide and kept in the direct sunlight, provided the sunlight is continuous and bright. In the other jars, only a slight amount of oxygen accumulates, depending on the quantity of direct sunlight and of carbon dioxide present. The results are highly gratifying.

The leaves in the various jars can be tested for the presence of starch in the usual manner. The results are very satisfactory.

FULLER'S EARTH.

The Geological Survey's annual statement on fuller's earth is now available for distribution. According to this report the production of fuller's earth during 1915 increased 6,920 short tons in quantity and \$85,573 in value over 1914.

**THE PRESENT STATUS OF ZOOLOGICAL TEACHING IN
MICHIGAN HIGH SCHOOLS.**

By HAROLD CUMMINS,

*University of Michigan, Ann Arbor.**(Concluded from the December, 1916, issue.)*

Field work is a required part of the course in about half the schools studied. Usually, only one or two trips are made, and these are for vertebrate studies in the spring and for insects in the fall. Some voluntary trips are encouraged, and in a few schools the pupils collect local animals for use in their own laboratory study. A few schools in the cities and larger towns are able to conduct visits to public museums and aquaria, but through lack of time find it impossible to reach a good locality for field study, to make observations, and to return within the limits of even a double period. Sometimes local climatic conditions render field trips unproductive. It is here suggested that while field work can best be done in the suburbs or country, most city schools are within reach of vacant lots, gardens, lawns and parks where some kind of outdoor work can be carried on. Of course, this would necessitate changing the character of field work, the kinds of animals sought, etc., but even these limited spaces will permit the finding and study of living animals of one sort or another in their natural environment.

The desirability of demonstrating specimens of the animals discussed in the laboratory and classroom is recognized by all teachers, yet few schools are provided with permanently preserved demonstration material. Of thirty-six schools, sixteen report no collections. The others, with three or four exceptions, possess a few birds and mammals and preserved representatives of the lower forms. In the laboratories which the writer has visited, and which have collections at all, there have been good series of marine invertebrates purchased from supply houses, and a surprising lack of local forms. A collection is of value in so far as it increases the pupils' knowledge of animals as they affect the pupils' everyday lives and supplements their formal school study of zoology. It should contain at least a fair representation of animals occurring in the locality. A school with an established course can offer no excuse for the lack of such a collection. Since pupils constantly bring in animals of all sorts, and boxes, bottles, fruit jars and formalin cost so little, there is no reason for not saving these specimens. The teacher can utilize the collecting interest in building up a small collection. He can secure

and retain further interest by allowing pupils to prepare specimens. In one laboratory, such preparations are made by the pupils themselves, with the collector's name included on each label.

The phases of zoology emphasized in both classroom and laboratory, arranged in the order of their most frequent occurrence in answers, are: Economic phase, structure, ecology, physiology, evolution, scientific method, interrelations of organisms, "practicalness," classification, habits, wonder of human structure, identification, and sex hygiene.

Answers dealing with laboratory work alone have yielded more definite results. The courses in twenty-six schools fall into two classes, viz.:

1. Primarily structural studies—twenty-one schools.
2. Combination of structure and studies in dynamic zoology—five schools.

Eleven additional answers are grouped as follows:

1. No laboratory—two schools.
2. Answers too incomplete for classification in the table just above; they probably belong, however, in Class 1 of that table—nine schools.

Extracts from typical answers will demonstrate our basis of classification:

1. PRIMARILY STRUCTURAL STUDIES.

(a). "We take up examples of the more important types and spend most time on the common ones with which the student is going to come most in contact. In this work, we dissect for the various systems (grossly) and give considerable attention to structural differences."

(b). "Types used, in order: Locust, June beetle, Monarch butterfly, house fly, amoeba or paramaecium, fresh-water sponge, hydroid, starfish, sea urchin, earthworm, fresh-water mussel, crayfish, frog. Character of work: 1. External plan of structure; (2). Appendages (uses); 3. Internal structure (parts and uses), digestive organs, respiratory organs, circulatory organs, nervous organs, reproductive organs, muscular system; drawings and notes for each exercise."

2. COMBINATION OF STRUCTURE AND STUDIES IN DYNAMIC ZOOLOGY.

(a). "We do not make a study of types to represent different phyla. Forms are chosen to illustrate certain groups as insects, worms, protozoa, etc., or to show certain methods of living as parasites, or for economic values as birds and fur-bearers. . . . Many living animals are kept in the laboratory during the year and much work is done in the study of the living form. . . . Not much work on internal anatomy is given. If students express a wish to see the 'inside' of any animal, they are allowed to dissect it, or a demonstration is given. The year's work is intended to give an acquaintance with the more common animals of this vicinity."

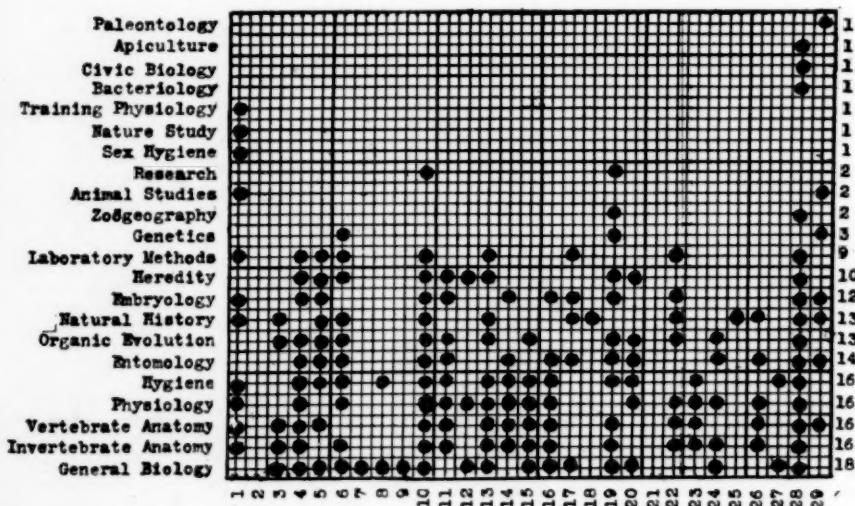
The writer, in common with others, holds that this undue emphasis on structure is the chief weakness of high school zoology. Without question, some structural studies are necessary, but they should not supersede other phases of the subject which are of more vital interest and more intimately fitted to the

needs of high school pupils. Nor are purely structural studies fitted to the psychology of adolescents.

That such teaching is only a reflection of the teachers' training is borne out by data received from Questionnaire II. Reference to Chart II will show that some teachers have been very inadequately trained, in fact two have not pursued course work in the subject. It is impossible to believe that such untrained teachers are fitted to present a fair, well-rounded view of zoology. Except in the rare case where the teacher has been efficiently self-trained, it would seem advisable to omit this subject from the course of study until such time as a well-trained teacher can be secured. In other cases, there has been pronounced morphological bias to the training. Deficiencies in this respect are recognized by many teachers, as evidenced by their answers to this question: "Do you feel that your preparation in college was of the sort which enables one to give the proper sort of high school course?" The opinions of individual teachers on the value of their course work are presented in the following almost verbatim extracts. Since it may be of importance to know the name of the institution in which each prospective teacher studied, that information is also given. To facilitate comparison, each teacher is assigned the same number in the extracts and on Chart II.

EXPLANATION OF CHART II.

At the left margin, the courses are arranged in the order of frequency with which they occur in the lists submitted by twenty-nine teachers.



Distribution of courses elected by twenty-nine teachers.

nine teachers. At the right is a summary of the number of teachers who elected each course of study. Each number on the base line represents a teacher, and the dark circles above that number indicate his elections.

OPINIONS OF TWENTY-NINE TEACHERS OF THE VALUE OF THEIR COLLEGE TRAINING IN ZOOLOGY.

1. Ypsilanti Normal. "Yes, have been prepared to meet many of the problems coming up outside of textbook used."
2. ? "I prepared myself for teaching zoology. In one way I consider I am better prepared to teach the subject, for I have observed the work of several college graduates who inflict on the young students the impractical kind of work that is adapted to college students."
3. U. of M. "No, not enough field and natural history courses."
4. U. of M. "I feel that my college course was deficient in field work."
5. U. of M. "Would have liked more field work, physiography and geology. Would omit pedagogy, and substitute special field studies."
6. U. of M. "Feel that my own course is successful after having had four years' specialization in the natural sciences and practice teaching."
7. Alma. "Yes, it gave the material which is of use in high school teaching."
8. U. of M. "Yes, studied much outside of school."
9. Central College. "Yes, a high school course must of necessity be general."
10. Ohio Wesleyan. "I hope so, I tried to prepare myself for teaching."
11. Kalamazoo College, U. of M. "No, needed work in laboratory methods."
12. C. S. N. S.² U. of M. "No, did not make a specialty of the work."
13. Valparaiso. "Yes, took enough to become interested myself, and to arouse interest in the students."
14. M. A. C.³ "Yes, obtained a broad and technical training, and have a natural fondness for animals."
15. W. S. N. S.⁴ "Yes, such training is fitted for the kind of course given in this school."
16. Hillsdale College. "Yes, covered the ground fairly well, but would have liked less anatomy and classification and more economic zoology."
17. U. of M. "Yes, what I had was good, but had to be supplemented by more recent study."
18. U. of Chicago. "Yes, only had too little of it. Specialized largely in botany, not realizing that general biology would be required of a high school teacher."
19. U. of M. "Yes, but would have liked a course in the natural history of vertebrates."
20. M. A. C. U. of M. ?
21. U. of M. "No, had not studied zoology in the university."
22. Denison Univ., U. of M. "Yes, had good foundation both in laboratory and field study."
23. W. S. N. S. "Yes did practice teaching; would advise acquaintance with a number of elementary texts."
24. W. S. N. S., U. of M. "Not entirely satisfactory; would have liked more study of living forms."
25. N. S. N. S.⁵ W. S. N. S., U. of M. ?
26. Kalamazoo College, U. of Chicago. "Yes, am able to carry out the same methods here, but after a simpler fashion."

¹U. of M., University of Michigan.

²C. S. N. S., Central State Normal School (Michigan).

³M. A. C., Michigan Agricultural College.

⁴W. S. N. S., Western State Normal School (Michigan).

⁵N. S. N. S., Northern State Normal School (Michigan).

27. U. of M. "Yes, believe that there is a danger of knowing too much."
 28. Ohio State. "Yes, enabled me to do research work."
 29. Ypsilanti Normal U. of M. "On the whole, yes."

When we realize that the teacher does not commonly teach biology alone, but must prepare himself to handle additional subjects often not related to biology, it is evident that his training in any one field must of necessity be limited. If the prospective teacher in his collegiate work cannot elect many courses in each of several fields, it is imperative that they be of a nature which will best fit him for presenting the subjects to high school pupils. Suggestions as to proper preparatory work in zoology made by those Michigan high school teachers now in service, who are fitted to advise on the character of courses needed, would probably result in desirable changes, at least in the State University. Reference to the following table will demonstrate the range of subjects taught in addition to zoology. In this table, each group represents the subjects taught by a single teacher, and the number following each group is the enrollment of that school. The data are from thirty-six schools.

CLASSES TAUGHT IN OTHER SUBJECTS.

Zoology alone.	Botany.	Botany.
2,500.	Physiology.	Physiographry.
	670.	History.
Botany.	Botany.	Algebra.
?	Physiology.	200.
Botany.	Physiography.	Botany.
250.	Com'l geography.	Latin.
Botany.	240.	175.
700.	Botany.	Botany.
Botany.	Physiology.	English.
700.	Physics.	75.
Botany.	Chemistry.	Botany.
750.	233.	Mathematics.
Botany.	Botany.	86.
930.	Physiography.	Arithmetic.
Botany.	562.	Business.
Physiography.	"All science."	1,400.
Physics.	165.	Botany.
150.	Botany.	Chemistry.
Botany.	Horticulture.	Physics.
Physiography.	Elementary science.	150.
Physiology.	?	Botany.
Physics.	Botany.	Physiography.
Chemistry.	Physiography.	U. S. history.
150.	Com'l geography.	240.
Botany.	Com'l arithmetic.	Botany.
Physiology.	274.	Mathematics.
		80.

350.	Botany.	Botany.
"All science."	Physiography.	Chemistry.
320.	U. S. history.	Physics.
	250.	Bookkeeping.
		110.
Botany.	Botany.	Botany.
Physiology.	Mathematics.	Physiography.
Physiography.	76.	Physiology.
1,183.	Botany.	Chemistry.
Botany.	Geometry.	250.
Physiography.	Arithmetic.	
Physics.	Phylography.	
Chemistry.	118.	
150.	Botany.	Botany.
Botany.	Physiography.	Physiography.
Physiography.	English.	Geometry.
588.	300.	Algebra.
		Literature.
		90.

We have already considered the importance of the teachers preparatory training in determining the character of their own instructing. But a factor not to be overlooked is the university entrance requirements. Despite the fact that the majority of high school pupils never enter college, many schools prepare all their pupils to meet the requirements for university entrance. The old requirements for entrance credit in the State University rendered obligatory so much structural work that many teachers had time for little else. But since the change of last year, this element has been eliminated in so far as the greater freedom in selecting material is concerned.

OLD REQUIREMENTS FOR ENTRANCE CREDIT IN ZOOLOGY, UNIVERSITY OF MICHIGAN, AS APPEARING IN CATALOG FOR THE YEAR, 1914-1915.

Zoology.—An applicant who offers a unit in zoology will be expected to have a knowledge of at least eight of the following animal types: 1 and 2. Two protozoa: Amœba, Paramecium, Vorticella, Stentor, Volvox; 3. A sponge: Spongilla or Grantia; 4. A hydroid: hydra to be compared with a medusoid form; 5. An echinoderm: starfish or sea urchin; 6. An annelid: the earthworm or the leech; 7. A crustacean: crayfish, lobster, or crab; 8. An insect: butterfly (including immature stages), grasshopper, cricket, cockroach, or other insect; 9. A mollusk: the fresh-water mussel or one of the snails; 10. A fish: minnow or perch; 11. An amphibian: frog, tree toad, toad, salamander (*Ambystoma*), or mud puppy (*Necturus*).

These forms must be studied by the laboratory method. Laboratory work should be directed, not merely toward a study of animal structure, but as far as practicable toward the study of habits and reactions. It should furnish the basis for the classroom discussion of principles; especially of evolution. Of the four periods per week that must be given to the work, two at least should be given to recitations or other class exercises. Careful original notes and drawings must be presented by applicants as part of the examination.

The mention of the following books may serve to indicate the character of the work required: NEEDHAM'S *Elementary Lessons in Zoology*; DAVENPORT'S *Introduction to Zoology*; JORDAN AND KELLOGG'S *Animal Life*; FRENCH'S *Animal Activities*.

REVISED REQUIREMENTS FOR ENTRANCE CREDIT IN ZOOLOGY, UNIVERSITY OF MICHIGAN, AS APPEARING IN CATALOG FOR THE YEAR, 1915-1916.

Zoology.—The unit or half unit in zoology must include laboratory and field work as well as classroom exercises. Wherever possible, two classroom periods should be available for each field or laboratory exercise. Two or three such exercises with two or three recitations per week make a suitable distribution of time.

The content of the course should be determined in some measure by local conditions, such as size of class, accessibility of suitable conditions for field work, and training and interests of the teacher. Study of types is essential, but these should be selected largely from the local fauna. Such study need not take up the details of internal structure nor require dissection by the student. As far as possible, the course should deal with living animals and should emphasize the functions, activities and relations to environment of the types selected rather than their morphology. But the study of types should serve merely to introduce the student to the local groups to which they belong. The outlines of the classification of these groups, recognition in the field of their common local representatives, their habits, life histories and ecology should form the larger part of the course. Emphasis should be placed on facts and principles that have a peculiar local interest and on the economic phases of the subject. In the small towns, field work and topics related to agriculture may be emphasized. In the cities, the work may have to be conducted largely on types and in the laboratory but types of economic importance should then be selected and the zoological aspects of civic biology given special attention.

The following textbooks are suggested: NEEDHAM'S *Elementary Lessons in Zoology*; DAVENPORT'S *Elements of Zoology*; JORDAN AND KELLOGG'S *Animal Life*; LINVILLE AND KELLY'S *Textbook in General Zoology*; HEGNER'S *Practical Zoology*; HUNTER'S *Civic Biology*; PEABODY AND HUNT'S *Elementary Biology*; KELLOGG'S *Animals and Man*.

Need for a training school at the State University which will include practice teaching in high school subjects is only too obvious. With the present methods of collegiate instruction, the prospective zoology teacher seldom learns how his subject should be presented to young people. He is graduated, and goes out to teach; he employs the same methods and the same subject matter which his own college instructors employed.

In conclusion, the writer wishes to make a plea for vitalizing zoology, for dynamic studies rather than strictly static studies in anatomy, and for more study of living animals in laboratory and field.

Acknowledgment is due to Professor George R. La Rue of this University for assistance and kindly criticism throughout the progress of this study, and to the teachers who cooperated by supplying data.

THE FUTURE OF CHEMISTRY IN THE HIGH SCHOOL.

BY ROBERT H. BRADBURY, A. M., PH. D.,

Head of the Department of Science in the South Philadelphia High School for Boys.

(Concluded.)

(c). Limitations, in actual work, of the principles just discussed.

I recall a fellow student in high school who, during recess, offered a criticism of the teaching of geometry which ran somewhat as follows:

"If these theorems are any good, we ought to learn as many of them as possible in the time we have for the subject. Now we could learn ten times as many, if we did not bother with the demonstrations. The demonstrations are useless anyhow, for we are all willing to believe the theorems without proof, and we ought simply to learn them by heart."

The fallacy in the recommendation of this young educational reformer is, of course, that drawing inferences is the main business of life, the one thing in which we are all constantly engaged, but his proposal embodies an effective caricature of the present plan of presenting chemistry by trying to nurture the mind of the beginner on an artificial product composed of the results of the science, with little reference to the method by which they are obtained. School geometry is real geometry, as far as it goes, and school Latin real Latin, but school science, conducted on this plan, is not real science at all, but a mere popular survey, from which the essential scientific spirit is absent. This is the main reason that the sciences have not accomplished what was expected of them when they acquired an assured place in the curriculum.

The future is sure to change this state of things. When tradition and inertia have been overcome, and the schools begin to render their service to the community with maximum efficiency, the subjects which stand in intimate relation to life and its needs will dominate the curriculum, and the ballast which we have inherited from a former age will occupy a subordinate position, or even disappear.

We have just examined two principles with which our teaching must, sooner or later, be brought into agreement. Let us now attempt to forecast the limitations which will be encountered when

we attempt to apply these generalizations in actual teaching in the American high school.

(a). The plan just stated, and worked out in a typical study of the atmosphere, must not be regarded as a rigid Procrustean scheme, into which every topic must willy-nilly be forced, but as an ideal, to be approached quite closely in some cases, and to be departed from widely in others. The sulphides, air and water can be handled in this way with admirable results. The next topic which naturally suggests itself, that of common salt, offers more difficulty, on account of the impossibility of extracting the sodium from sodium chloride in the laboratory, but the method can still be used with great profit. With fluorine and its compounds, or with the hydrocarbons, the plan would be so unsuitable that sane common sense at once dismisses it as impracticable. Here we must resort to communication, vitalized by reference to familiar phenomena, by the mineral collection, and by significant laboratory and demonstration experiments. We shall see, in a moment, that, in all topics, the treatment recommended must, at some stage or other, retire and give place to systematic communication.

(b). "Rediscovery" is not the object of the work, and is, in general, impossible under high school conditions. We are all acquainted with the futile and dangerous tomfoolery of the pupil who starts to investigate on his own account. Leadership there must be, and the dogmatic analytical method at its worst is far better than the anarchy of haphazard experimentation.

I insist on this elementary point on account of the exaggerated statements which are published from time to time by critics without experience in high school teaching. Claims that the student should himself select the problem, devise the method of attack, work out in tributary researches any general principles that may be needed, and construct the apparatus required, are sheer nonsense, which serve only to delay needed reforms and to provoke reactions to mechanical methods.

Take the particularly simple case of Boyle's law. The lad who was provided with a stand, meter stick, glass tubing, mercury, and *no guidance*, would have about the same chance of arriving at the law as a lot of type would have, when placed in a bag and shaken, of setting up the text of *The Tales of Baron Munchausen*. *With* guidance, which tells him exactly what to do, and to all intents and purposes what he is to observe, he is able to understand the train of thought which Boyle pursued, and to verify the

law up to a pressure of about two atmospheres. Boyle himself went no further, though he did experiment with air at pressures of less than one atmosphere. This places the student in a very different position from that which he would have occupied if he had merely chanted a formula that the volume varies inversely as the pressure, but he has not *rediscovered* any general law about the conduct of gases. He has followed the track blazed for him by Boyle, and he has worked with but one gas and over a very small range of pressures. Communication must now be resorted to, if he is to know anything about higher and lower pressures, and about the behavior of different gases. Similar statements hold good with regard to the expansion of gases by heat.

The student's personal knowledge, even supplemented by the laboratory, is narrow, and must be enlarged by communication on every topic. But there is all the difference in the world between communication to the learner who has worked through concrete similar examples, and communication to the learner who has not, and who must try to find a foothold on the unsubstantial foundation of mere verbiage.

The idea of atomic weights does not become concrete so readily. Explanation is of hardly any service here—problems are better—but I have not found it possible to give a concrete working knowledge of this indispensable idea by problems alone. The tendency to confuse atomic weight with *density* is surprisingly difficult to eradicate. To let the student determine two or three atomic weights for himself at once annihilates his perplexities. The atomic weights of copper, tin and magnesium can each be determined with sufficient accuracy in about an hour, with only the apparatus at hand in every laboratory.

In this procedure of generalizing from a few instances only, there is nothing unscientific, for the actual investigator does exactly the same thing. No research worker tests all included cases before forming his generalization. He may often set up a working hypothesis on a basis of one instance only.

III. A WORKABLE SCHEME.

It is time for us to profit by the experience of some of the European nations which, while behind us in laboratory facilities, are in advance of us in the arrangement of the work. The course should be divided into two distinct phases, each with its own aims and purposes.

I. *The First Phase.*—The object of the first phase of the

chemical work is the orderly development of the general indispensable principles of the science. Method should be supreme, chemical system and classification should be allowed no weight whatever, and the condition of the mind of the student should not be forgotten for a moment. Each topic should be started with something concrete and significant to him, or with something which can at once be rendered concrete in the laboratory, or by the aid of the mineral collection. A typical example of this kind of treatment has been sketched in connection with the atmosphere. Everything except scientific accuracy should be subordinated to the observance of the two principles already discussed. As subjects for this first course, the following naturally suggest themselves:

(a). A study of familiar elements which occur native, especially sulphur, carbon and the common metals. This should include the mineral sulphides and the preparation of the metallic sulphides, and out of it should emerge the essential characteristics of a chemical process.

(b). A study of the atmosphere, somewhat as indicated above, which should be extended to cover combustion, carbon dioxide, etc. If not relegated to physics, the general properties of gases and the kinetic theory of matter—which is now a fact—can form the conclusion.

(c). Water and hydrogen, the latter being obtained not from acids, but from the water by means of zinc dust. A natural conclusion is formed by the action of hydrogen upon oxides and of carbon upon oxides, with the related metallurgical processes.

(d). Common salt, sodium and chlorine. Hydrogen chloride in detail and briefer treatment of the important compounds of chlorine with the elements already studied. The first phase can here be concluded with a concise, systematic survey of the ground thus far covered.

Symbols and formulas can be introduced in (b) in connection with the oxides of the metals, the sulphides being also pressed into service. The idea of molecular weight can be adumbrated in (c) and fully set forth in (d). The essential facts of solution can be given at the beginning of (d) in connection with the physiographical and industrial treatment of salt, but the application of the idea of molecular weight to solutions, and the treatment of ionization and electrolysis are best deferred.

II. *The Second Phase.*—In the second phase, also, the two great educational principles which dominate the whole should

receive constant attention, but, since the student now has a considerable chemical experience, and quite a large store of familiar instances with which to assimilate his new acquisitions, the movement can be freer and more systematic, with somewhat less emphasis on methodological considerations. Space is lacking to enumerate the subject matter, which includes the chemical domain which every child should know. I must confine myself simply to making a few suggestions.

(a). If time is available, the brief summary indicated as a suitable conclusion to the first phase can be omitted, and replaced, at the beginning of the second phase, by a more detailed and systematic study of some of the more important elements which have already been encountered. This would be the place, for example, for the production of hydrogen from acids and of oxygen from potassium chlorate, for ozone and hydrogen peroxide, and for further details regarding the chlorine compounds.

(b). Consider a sequence of substances like the following: Lead oxide, lead sulphide, lead chloride, lead sulphate, lead nitrate, lead carbonate, etc. The only connection is that all the seven materials contain lead. In the study of such a series, there is an incoherence, a jerky mental movement, due to the inconsecutive-ness of the arrangement, to the almost complete lack of congruity among the topics. In a handbook, such a grouping is satisfactory, because it makes it easy to find things. But there is no good reason for copying the arrangement in the presentation of a science to the beginner. Familiarity, logical order and congruity are the only considerations which are relevant and to which any weight should be attached. Lead sulphide, as an important mineral which can be cheaply purchased in almost pure condition for laboratory work, finds its natural place among the sulphur compounds, near the beginning of the first phase. Lead oxide can be logically handled in connection with the action of air on the metals.

Where some systematic classification must be followed, it is usually better to group salts according to the acid radical, rather than according to the metal, for the purposes of elementary study. Thus, the nitrates form a fairly coherent topic, closely connected genetically with nitric acid and with the oxides of nitrogen. The solubility relations of the nitrates can be discussed in the light of the important case of potassium and sodium nitrates, and the fixation of nitrogen forms a suitable conclusion. All this cohesion is sacrificed if the nitrates are merely scattered among the metals.

(c). I have elsewhere expressed the opinion that the actual value of technology to the student is greatly exaggerated in some quarters. He will more frequently have opportunity to apply the purely scientific aspects of the subject. Moreover, through no fault of the teacher, the technology of the high school course is apt to diverge quite widely from the actual conditions in practice, which change so rapidly that the processes of the elementary text are often obsolete before the book appears in print. All of us are acquainted with the constant necessity of correcting the technology of the book we happen to be using.

As an illustration of the uncertainty of technological information, consider one point in a rather stable process—the manufacture of common soda glass. In common, I suppose, with most teachers, I have been in the habit of dismissing the sodium aspect of this matter with the statement that the carbonate had been used but was largely displaced by the sulphate, which furnished the sodium more cheaply. Two years ago, I learned, in conversation with a large glass manufacturer, that Chili saltpeter had almost entirely displaced the sulphate, the oxidizing action being regarded as an advantage. At present, while the imports of Chili saltpeter into the United States have not increased to any great extent, the demand for it for the nitric acid manufacture is quite unprecedented, and the glass makers seem—to judge from that uncertain criterion, newspaper reports—to be forced back to the use of soda ash.

The sane thing seems to be to add interest by the aid of technology, to keep the applications of chemistry to human needs before the students, but to avoid overemphasis. Special attention should be paid to those cases in which a technical process exemplifies and clears up an important general principle. An illustration is the connection of catalysis with the manufacture of sulphuric acid, both in the lead chambers and by the contact process.

(d). In all these recommendations there is nothing revolutionary and nothing radically new. In Germany, the abandonment of the dogmatic analytical procedure began with Arendt (1862) and Wilbrand (1870), and has been complete for a generation. In England, the same movement has progressed under the leadership of Perkin and Armstrong. Since the basic idea is to progress logically from the concrete to the abstract—and, where possible, to return to the concrete at the end of the train of thought to clinch the principle by an application significant to the

student—the plan leads to an unusual attention to household matters, familiar phenomena, everyday affairs. Thus far are we in accord with those who would have us make our teaching more practical. But we wish to interpret the word, not with the myopic vision of the half-educated man, who despises everything which he cannot understand, but with the wide and imaginative outlook of the man who knows that the disinterested research of today is the factory commonplace of tomorrow. The commercial history of the world for the last fifty years points the moral that the nation which fails to grasp the value of science, and especially of chemical science, is moving to disaster. America should learn this lesson *now*, and not through an industrial catastrophe hereafter.

OUTLINE FOR THE STUDY OF BITUMENS.

This is arranged in convenient question and answer form, with space for additional memoranda, and is published by The Barber Asphalt Paving Company. While the whole subject of bitumens is covered, the *Outline* has been prepared with especial reference to the asphaltic materials used in highway construction. In addition to the answers provided in the *Outline* itself, there are references to most of the standard textbooks on highway engineering. While prepared especially for school use, the *Outline* is a convenient means of reference for anyone who finds it necessary to investigate the differentiation and characteristics of bitumens.

FOSSIL PLANTS OF GREAT AGE.

The area known as the Mississippi embayment is a low-lying region which has alternately been submerged and emerged since Cretaceous time—known as the Age of Reptiles—over five million years ago. It embraces roughly 1,500,000 square miles. In past geologic ages, this region doubtless furnished congenial habitats for several thousand specific types of plants of which we can never hope to know more than a small number. Nearly all these types have irrevocably vanished, and this vast area is tenanted today by an entirely new set of plants. The United States Geological Survey, Department of the Interior, has recently published a scientific report by E. W. Berry, describing some of these long-vanished plants, the fossil remains of which are found today in many of the rock formations. The report gives a systematic description of more than three hundred species, in what are known as the Wilcox and Midway formations.

The Wilcox flora is made up almost entirely of plants that lived along the ancient coast, on the strand, among lagoons and sand dunes. The physical conditions under which the plants lived are discussed, and conclusions are drawn regarding the climate of the Eocene period compared with that of today in the same region.

A copy of the report (*Professional Paper 91*) may be had free on application to the Director of the United States Geological Survey, Washington, D. C.

MODERN DEVELOPMENTS IN ELEMENTARY AND SECONDARY MATHEMATICS.¹

BY G. A. MILLER,
University of Illinois.

Mathematics is not indigenous to America like potatoes, tobacco and Indian corn. In fact, mathematics is not indigenous to any country in the sense that it was found in a highly developed state in a certain country, and from there spread over the world. Like the human language, our subject seems to have taken root wherever human beings congregated. This subject is itself largely a language relating to certain intellectual apperceptions which are fundamentally connected with society, even in its primitive forms.

Mathematics is essentially a universal language and the mathematical symbols have taken the lead in proclaiming to the world the folly of various written languages. The Hindu-Arabic numerals are with us as messengers from almost our antipodes to direct our attention to the fact that, in abstract records relating to counting and measuring, it has been proved that we need not exhibit any differences. Why should we then insist on differences relating to the concrete? If the equation,

$$x^2+3x-2 = 0,$$

can travel around the world without twisting itself into different forms for the sake of being understood, why should not a printed sentence have the same advantage?

While mathematics is not indigenous to any particular land, and while our subject tends towards universality, these are general observations that become much clearer if attended by modifying details. In the first place, special mathematical developments are to a large extent local products, and most of them have radiated from a single center like light and heat coming from the sun to bless the worlds of our solar system, and perhaps to brighten the skies for other worlds. Although mathematics as a whole is not indigenous to any country, it might be said that many special developments have this property, so that our subject may be regarded as composed of many lights of various intensities and coming from various countries.

Mathematical history largely concerns itself with seizing the rays of mathematical light and with following them back to their apparent sources. Behind these apparent sources, there lie hid-

¹Read before the Kansas Association of Teachers of Mathematics, November 10, 1916, and before the Mathematics Section of the Central Association of Science and Mathematics Teachers, December 2, 1916.

den the truer sources bound in an apparently impenetrable maze. Just as we may follow a sunbeam to the sun and then stop, our mathematical history may lead us to a certain original source and then stop. In both cases, the question of greatest interest is only begun at this point. Whence came the energy which produced this sunbeam, and whence came the intellectual forces which prepared the way for the development exhibited by these original sources?

American mathematics has been profoundly influenced by foreign developments. During the second half of the preceding century, a large number of Americans began to go to European universities for the purpose of studying mathematics. Most of the members of this colony of frontiersmen went to Germany, and hence it is of especial interest to inquire what they found there at that time, since this will help us to understand what kind of influences they spread among us.

We find that German mathematics during the greater part of the second half of the nineteenth century was getting more and more abstract and formal. It was not until about 1890 that a strong movement was inaugurated to lay more stress on applied mathematics in Germany.² This movement naturally made itself felt somewhat slowly, and many of our young, ambitious American mathematicians returned from Germany, even after this date, with the laudable spirit of free search in any field of pure mathematics, but too few of them were also imbued with the need of keeping in touch with applications. Although most of these men entered university faculties, yet their teaching and example influenced deeply also our elementary and secondary mathematics.

Hence the cries, "Back to applications," "More concrete mathematics," and "Emphasis on graphical methods," were a natural outcome of an undue earlier emphasis on pure mathematics. These cries had been heard many times before, but recently they became unusually persistent and world wide. At least one of the cheer leaders, John Perry, has already won international fame as a result of this wave of enthusiasm. Today there are many others who would like to win fame by going to still greater extremes, but some of them seem to forget that even the cheering crowd maintains some sense of proper proportions which the successful cheer leaders cannot afford to overlook.

In a very broad way, it should be said that the recent develop-

²P. Zühlke, *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht aller Schulgattungen*, Vol. 45 (1914), p. 483.

ments in elementary mathematics have been largely influenced by an emphasis on applications, and this may serve to explain the great progress during the last few decades towards the earlier use of differential and integral calculus, and the introduction of the elements of trigonometry into our high school courses in geometry. Trigonometry means measuring the triangle, while geometry means measuring the earth, and it is absurd that we should try to measure the earth before we learn how to measure the simplest of the figures on the simplest surfaces.

Some one may be inclined to reply that this is merely a play on words which lost their original significance long ago, but further thought will tend to reveal the fact that the relative intrinsic difficulties in elementary geometry and elementary trigonometry, as they are often presented in modern textbooks, are fairly well represented by the complexities of the surface of the earth and the simplicity of the triangle, respectively. If the elements of trigonometry are to be taught as a separate subject, they should clearly be taught earlier than the abstractions of our elementary geometry. The calculations involved in elementary trigonometry have close contact with those of arithmetic and algebra.

It would probably be best not to teach trigonometry as a special subject since it is merely a part of metrical geometry. According to the comparative study of courses in the countries represented in the International Commission on the Teaching of Mathematics, published in 1915 under the title, *Curricula in Mathematics* by J. C. Brown, "the introduction of the trigonometric functions while the pupil is studying similar figures in geometry has the sanction of most of the best teachers abroad." These functions are so useful and so constantly met in the mathematical literature that those who do not go beyond the high school course in our subject should be familiar with them, and those who go to the university cannot become too familiar with notions which are so useful, not only in the study of mathematics, but also in the study of physics, chemistry and other sciences.

In order to present to you with definiteness some modern developments in elementary mathematics, it is necessary to understand what is implied by the term, "elementary," in connection with our subject. A common notion about elementary mathematics is individualistic and is based on the assumption that everything is elementary that we understand, and what we do not understand is advanced. Such a conception may be more or

less natural, but it cannot very well be made a basis of classification. It is, however, not very different from the modern tendency to call elementary whatever is simple and fundamental, or to regard elementary mathematics as the alphabet of our subject.

It is interesting to note that the Greeks used already the term, "elementary," in connection with mathematics, and Euclid's *Elements* have, in fact, stood for the elementary part of mathematics for nearly two thousand years. It is true that elementary trigonometry and elementary symbolic algebra were largely developed during this period, but these subjects were not classed with the elements of mathematics until a later period. The suggestions that the concepts of limit and infinite processes should be avoided in elementary mathematics are scarcely feasible, since these concepts enter into some of the most interesting and useful work of elementary algebra and geometry, such as finding the limit of the sum of an infinite geometric series whose ratio is less than unity, or finding the area of a circle.

According to F. Klein, elementary geometry is the geometry of the principal group. That is, the group composed of all movements, similarity transformations, reflections, and their combinations, which is represented by the following equations of transformation,

$$\begin{aligned}x &= \lambda(a_{11}x' + a_{12}y' + a_{13}z') + b_1, \\y &= \lambda(a_{21}x' + a_{22}y' + a_{23}z') + b_2, \\z &= \lambda(a_{31}x' + a_{32}y' + a_{33}z') + b_3,\end{aligned}$$

if rectangular Cartesian coordinates are employed and the matrix of the coefficients of x' , y' , z' determines a proper orthogonal substitution. This furnishes a clear-cut definition of elementary geometry, but it evidently involves many developments which are not usually classed with this subject.

Although it seems impossible to give a satisfactory definition of elementary mathematics, it is clear that simplicity and utility should always be fundamental factors in determining whether a special subject or part of a subject should be classed with elementary mathematics. An important notion of elementary mathematics may be exhibited by denoting it as *middle mathematics*. The simplest things in mathematics often lie in the midst of the subject and not very close to the beginning thereof, if indeed it has a beginning. In elementary mathematics, we seek the truth, nothing but the truth, but not the whole truth. Much of

our difficulty, especially in elementary geometry, is due to the fact that we strive too often to begin near the beginning. The abstractions relating to postulates and their independence are usually very unsuitable for the beginner. It is entirely possible that the elementary geometry of the future will be the projective geometry of today, and that the elementary algebra of the future will deal with the notions of groups, invariants and derivatives.

The derivative is perhaps the most maltreated mathematical concept at the present time. Although it established a long time ago its right to dominate in much of our elementary mathematics, this right remains still unconceded by many of our modern tyrants in elementary mathematics, the textbooks. The tyranny of textbooks has been largely mitigated in recent years by societies and conferences of this kind, where teachers can voice their experiences in an effective manner, but there is still entirely too little supervision over this tremendous conservative force affecting needed reforms in elementary and in secondary mathematics.

Among the most interesting elementary problems which can be used to illustrate the derivative are those relating to the maxima and minima values of polynomials in one unknown. For instance, in many of our algebras, the maxima and minima values of quadratic functions of x are found by completing the square. In particular, if the minimum value of

$$x^2 + 4x - 2$$

is desired, it is often written in the form,

$$(x+2)^2 - 6,$$

and it is noted directly that -6 is the minimum value of this function, since $(x+2)^2$ cannot be negative.

On the other hand, if the degree of the polynomial exceeds two, this method is not, in general, directly applicable. Why should we teach such a special method when the derivative method is so much more general and is not difficult in theory, especially when we restrict ourselves to polynomials? Pretty special methods have a place in the later work of the student, but, whenever it is feasible to do so, the most general methods should be taught first. The general appreciation of mathematics is largely dependent upon the methods employed in the early mathematical work, and subjects of as wide applicability as the derivative should be taught as early as possible.

The two principal questions discussed at the meeting of the International Commission on the Teaching of Mathematics held at Paris in 1914 were: (1) The results obtained by the introduction of the differential and integral calculus into the higher courses of the secondary schools; and (2) the place and the rôle of mathematics in the higher technical teaching. The favorable reports relating to the use of the derivative and the integral at an earlier stage than formerly are doubtless known to many of you, as these reports appeared in the well-known journal, *L'Enseignement Mathématique*, during May and July, 1914.

Considerable progress has been made recently in the introduction of the concept of functions into our courses on elementary algebra. This does not mean that we are trying to teach an old-fashioned course of function theory to our beginners in algebra. It simply means that the elementary concepts of functions are introduced early and allowed to put more life into our algebra. Similarly, the introduction of the concepts of derivative and integral into our elementary courses in algebra and geometry does not imply that we should teach to the beginners in these subjects our present course in calculus. On the contrary, it means that the elements of the powerful calculus should be taught in place of such subjects as proportion, variation, determinants, continued fractions, finding the volume of a truncated pyramid, etc.

In matters relating to our physical comforts, we do not refuse, for instance, to ride in an automobile, electric car or a railroad train, because we do not understand everything about the mechanism of these conveyances. Why, then, should we refuse our intellectual lives the privilege of taking rides on conveyances which we do not fully understand? Much of our mathematical trouble results from the fact that we are working with the intellectual wheelbarrow and are compelling our students to work this way because they do not fully comprehend the intellectual automobile. The simple mathematical life has its charms, but it does not fit well into modern society.

Fortunately, there has been a growing tendency in recent years to prefer vigor to rigor in our elementary mathematics. The man of action needs a mathematics of action. The great war through which we are passing, at least in our sympathies and hopes, has already directed emphatic attention to the value of science and to the need of more thorough scientific training. The mathematics of the elementary and the secondary schools must fit into this scientific training, rather than try to turn the world

back to the philosophical speculations of the Greeks at the time of Euclid. The calculus, the function concept and the group concept relate to variations and action, and hence fit much more into the life of action of the modern man than the static propositions of Euclid, where even the opposition in direction now embodied in the negative sign is lacking.

While the idea of utility has recently revolutionized the teaching of elementary and secondary mathematics in many countries, and we are now in the midst of profound changes, it must not be forgotten that utility and abstraction are not necessarily antagonistic. In fact, abstraction is ideal utility, since it does at one time what would otherwise have to be done a large number of times. It is this highest type of utility which should animate our elementary instruction. The concrete applications should not be regarded as an end but as a means to secure greater and more tangible abstractions.

There is, of course, danger in proceeding too rapidly when we begin to make changes. As I understand it, the French introduced the study of the derivative in 1902 into their courses for students who were about fourteen or fifteen years old, but in 1912 this notion was placed about a year later. In our country, this notion is still commonly introduced as late as the sophomore year in the college, which seems certainly too late for its introduction. If the French and the German students can profitably begin the study and application of the derivative when they are fifteen or sixteen years old, the American students can do so too.

In Europe, the introduction of the infinitesimal and integral calculus into the secondary schools met with more opposition on the part of the professors in the universities than on the part of the teachers in these secondary schools.³ This opposition seems to have been based on the ground that if the student comes to the university with some notions of differential and integral calculus, he will take less interest in these subjects in the university than if he meets these very interesting concepts for the first time in his university course and has all at once a new mathematical world opened before his eyes.

This argument is of little value since most of the students do not have an opportunity to proceed beyond the high school, and, moreover, the high school does not exist for the sake of making the work of the university professor easy. I would much rather present to a student who has mastered the technique of arith-

³*L'Enseignement Mathématique*, Vol. 16 (1914), p. 298.

metic and algebra the delightful notions of analytic geometry and the calculus, than to have to contend with improvements in his technique in algebra. On the other hand, if the student has already the elementary notions of analytic geometry and the calculus, with a less thorough knowledge of algebra and elementary geometry, he can apply these notions in the study of other subjects, such as elementary physics, in which these notions could frequently be used to advantage.

The differential calculus furnishes the natural method of finding tangents and maxima and minima values, and the integral calculus furnishes the natural method of treating the length of curves, areas of surfaces, and volumes of solids. Fortunately, many of our textbooks on analytic geometry employ the differential calculus for the sake of finding tangents, but in many cases this is not yet done as openly as it should be done. The elements of differential and integral calculus really belong in the high school algebra course, and they should be openly introduced and called by their proper names. The highly interesting examples to which these elements could be applied would furnish a most appropriate chapter for those students who can go no further with their mathematical studies. They should, however, follow the introduction of the notion of function.

One trouble about our elementary mathematics is that the results do not sufficiently astonish the student by opening to him new realms of thought. The student is, as a rule, not much surprised to find that two plane triangles are congruent when two sides and the included angle of the one are equal, respectively, to the corresponding parts of the other. On the other hand we really open up broad and surprising vistas when we begin our elementary geometry course with an explanation of the fact that number pairs and number triplets may be placed in a $(1, 1)$ correspondence with the points of the plane and with those of our ordinary space, respectively, by means of systems of coordinates, and then show how geometry may be regarded as a picture book of algebra, how the equal values of a polynomial in one unknown lie on curves which pass into each other by translations, etc.

I often wonder whether the first chapter in our ordinary school geometry would be really any easier to the beginner than the first chapter in analytic geometry, if the latter were properly modified by the introduction of some notions of projective geometry. This could be followed by a study of a geometric interpretation of such simple groups of movements as the symmetric group of

order 6 obtained, for instance, by subtracting from unity and dividing unity, and the group of the square obtained by subtracting from 2 and dividing 2. Such very elementary examples would relate geometry with algebra in a most striking manner, and would furnish an important step towards the notion of group and group of transformation. In dealing with the elementary trigonometric functions, these groups would present themselves again in a natural way and would aid to simplify the considerations.

Perhaps elementary mathematics is best defined by saying that it consists in developing fundamental and prolific concepts of mathematics up and down, until we arrive at too serious difficulties. The fact that many recent mathematical developments relate to very elementary matters has often been emphasized. Hence, the teacher of elementary mathematics who would really be up with the times has the somewhat difficult task of keeping in touch with modern mathematical developments and to seek to appropriate such of these developments as seem most available for his work. This process of appropriating suitable elements of advanced mathematics for elementary instruction has generally been very slow, and hence one has to look over developments covering a number of years with a view to making the proper selections.

An important factor in elementary mathematics is the selection of a good notation. I have often regretted, for instance, the use of the square root symbol, $\sqrt{}$, with the double meaning of the positive square root of a positive number and also as the positive and negative square root of such a number. Various other symbols are used in such a way as to be ambiguous, as, for example,

3^{2m}

where the result is generally different if we regard this as 9 raised to the m th power or 3 raised to the 2^m th power. It is also unfortunate that we persist in using the decimal period when the decimal comma is more commonly used, and that the useless term "logarithm," which is equivalent to exponent, has not been banished. A noteworthy effort has been made in Germany in recent years to secure uniformity of notation in elementary mathematics, and I desire to direct your attention to an explanation of this effort, given in Volume 44 (1913-1914) of the *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, page 395.

In urging the earlier use of the concepts of our differential and integral calculus, it is, of course, not implied that more time should

necessarily be devoted to mathematics. In fact, we noted above various subjects which could well be placed later in our mathematical course than these useful concepts. I desire especially to emphasize the fact that the theory of proportion with its cumbersome algorithm should be eliminated from the courses in elementary and secondary mathematics, since the linear equation includes this theory and affords a much better and more uniform method of treatment. In fact, I see no reason why the subject of variation should not also be omitted since it, too, is included in the theory of the linear equation.

During the Middle Ages, they frequently taught various methods of multiplication, and they sometimes regarded doubling and halving as special operations. Our teaching the linear equation in the three forms of proportion, variation and linear equation is comparable, it seems to me, to the undue prolixity in some of the old textbooks. Our emphasis on the theory of proportion is probably due to the fact that this theory occupies such a prominent place in Euclid's *Elements*, but it should be noted that these *Elements* were written before the development of our modern symbolic algebra.

One of the most prominent among the new features of our modern textbooks is the increasing emphasis on historical notes, or notes which appear to be historical. In fact, some of the textbooks also introduce portraits. As far as these notes or portraits are based upon reliable sources, they are to be commended, but I would much rather see in an elementary textbook a picture of an automobile with the label, "This is the automobile Euclid would have used if he had lived in the twentieth century," than to see a portrait, labeled, "Euclid." The former would give just as much historical information as the latter and would have the advantage that it would not tend to deceive many students.

Portraits, based upon busts or inscriptions made hundreds of years after the time when the subjects of these portraits lived, are, historically speaking, nonsensical. In fact, those relating to the same subject sometimes differ so widely that they could not all be true. The available space for historical notes and portraits can be well filled with reliable and interesting data, and hence it appears the more deplorable that this space is so often used for what is worse than fairy tales, since it is intended for mature students.

In looking over the world's mathematical activities of today, we may see many modern movements of great interest. Prom-

inent among these are the publications inspired by the International Commission on the Teaching of Mathematics and the many modern organizations of mathematics teachers, including the Mathematical Association of America. The recent journals, in particular, the *Revista de Matematicas* which was started at Buenos Aires during last March, are signs of an unprecedented, widespread and growing interest in the various fields of our subject.

Our general attitude towards these modern developments should not represent the extreme of believing what is new is good and what is good is new. This extreme is, however, much better than the other extreme of thinking what is new is not good and what is good is not new. The true line of action lies clearly within these extremes, but the determination of its exact situation presents difficulties of a very serious nature. We can hope only for an approximate solution, but the approximation should become closer and closer as new light appears.

ROCKS MADE TO TELL THEIR OWN STORY.

The walls of the Grand Canyon in Arizona form a great natural geologic section, in which each layer of rock is in its original position relative to those above and below it. In few other places, however, is the story of the upbuilding of the earth's crust so plainly and impressively told. As a rule, the geologist who would decipher the records of the rocks must get a bit here and a bit there. He may find the edges of some beds exposed in a river bluff and others sticking out on a steep mountain side. He determines by fossils or other means the order in which the beds were deposited, and by putting all his information together he constructs what he calls a columnar section for the district in which he is working—that is, a section showing the order, thickness and character of the beds. Such a section discloses the strata that form the upper part of the earth's crust at that place, just as a slice of layer cake shows at a glance the various layers of which it is composed.

After a number of districts in a region have been studied and their general columnar sections determined, the geologic history of the region can be learned by comparing these sections, just as the engineer who is drilling for low-grade copper ores compares his drill records and thus learns the outlines of the ore body. Such a comparison of the beds at one place and another shows how certain beds change in character and thickness from place to place or even thin out and disappear. It enables the geologist to draw some conclusions as to the former distribution of land and sea, to distinguish the deposits laid down in deep water from those spread by rivers over their flood plains, and to reconstruct in imagination the course of events at a time long before the beginning of the Grand Canyon. Such a comparison has recently been made for Arizona and is published by the United States Geological Survey.

ANOTHER METHOD OF DERIVING SIN $2a$, SIN $3a$, AND SO ON.

BY NORMAN ANNING,

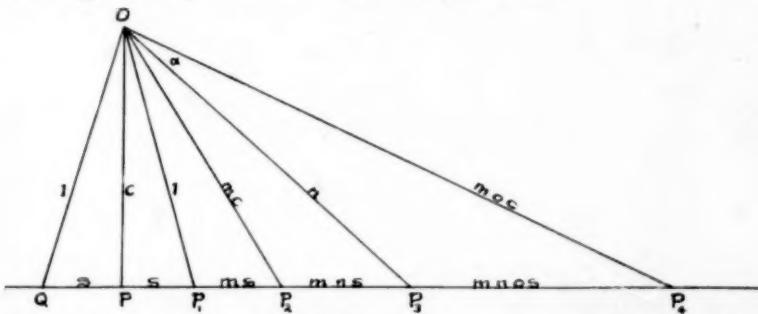
Chilliwack, B. C.

We assume the theorem that the bisector of the vertical angle of a triangle divides the base into segments that are proportional to the adjacent sides.

In the figure, OP is $\perp QP$ and angles QOP , POP_1 , P_1OP_2 , P_2OP_3 , and so on are each equal to a .

Choose OQ as unit length.

Then $OP = \cos a \equiv c$,
and $PQ = PP_1 = \sin a \equiv s$, and $c^2 + s^2 = 1$.



Since OP_1 bisects angle POP_2 , we may, using the above theorem, denote OP_2 and P_1P_2 by mc and ms where m remains to be determined.

Using the same theorem, we may denote,

OP_3 and P_2P_3 by n and mns ,

OP_4 and P_3P_4 by moc and $mnos$, and so on.

CASE I. THE DOUBLE ANGLE.

Since $\triangle POP_2$ is right-angled at P ,

$$m^2c^2 = c^2 + (s+ms)^2,$$

$$m^2(c^2 - s^2) - 2ms^2 - 1 = 0,$$

$$\{m(c^2 - s^2) - 1\}\{m+1\} = 0.$$

Only the first of these factors is of use to us:

$$m = \frac{1}{c^2 - s^2}$$

$$\cos 2a = \frac{c}{mc} = \frac{1}{m} = c^2 - s^2 = \cos^2 a - \sin^2 a.$$

$$\begin{aligned} \sin 2a &= \frac{s+ms}{mc} = \left(1 + \frac{1}{m}\right) \frac{s}{c} = (c^2 + s^2 + c^2 - s^2) \frac{s}{c}, \\ &= 2sc = 2 \sin a \cos a. \end{aligned}$$

CASE II. THE TRIPLE ANGLE.

Since OP_1 bisects angle QOP_3 ,

$$\frac{n}{1} = \frac{ms+mns}{2s}, \quad 2n = m+mn,$$

$$\begin{aligned} \frac{1}{n} &= \frac{2}{m} - 1 = 2c^2 - 2s^2 - 1, \\ &= 4c^2 - 2c^2 - 2s^2 - 1 = 4c^2 - 3. \end{aligned}$$

$$\cos 3a = \frac{c}{n} = 4c^3 - 3c = 4 \cos^3 a - 3 \cos a.$$

$$\begin{aligned} \sin 3a &= \frac{s(1+m+mn)}{n}, \\ &= s \left\{ \frac{1}{n} + m \left(1 + \frac{1}{n} \right) \right\} = s \left(\frac{1}{n} + m \cdot \frac{2}{m} \right), \\ &= s \left(\frac{1}{n} + 2 \right) = s(4c^2 - 1), \\ &= s(3 - 4s^2), \\ &= 3 \sin a - 4 \sin^3 a. \end{aligned}$$

CASE III. HIGHER MULTIPLES.

Since OP_2 bisects angle POP_4 ,

$$\frac{mo}{1} = \frac{mn+mno}{1+m},$$

$$o(1+m-n) = n, \frac{1}{o} = \frac{1}{n} + \frac{m}{n} - 1 = \frac{1-8c^2s^2}{c^2-s^2},$$

$$\cos 4a = \frac{1}{mo} = 1 - 8 \cos^2 a \sin^2 a.$$

$$\frac{1}{o} = \frac{1}{n} + \frac{m}{n} - 1.$$

Similarly, it will be found that

$$\frac{1}{p} = \frac{1}{o} + \frac{n}{o} - 1,$$

$$\frac{1}{q} = \frac{1}{p} + \frac{o}{p} - 1, \text{ and so on.}$$

This recurring formula enables us to extend the investigation at will. The geometric proof and interpretation, and the proof of the following theorem are left as an exercise. If the vertical angle AOE of a triangle be quadrisectioned by lines which meet the base in B , C , and D so that A , B , C , D , and E are in alphabetical order, then the segments of the base satisfy the following relation:

$$AB \cdot CD \cdot CE = BC \cdot DE \cdot AC.$$

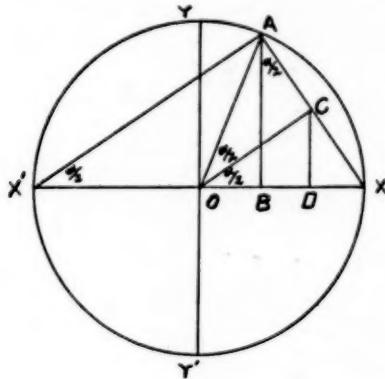
**GEOMETRICAL PROOFS OF THE FORMULAS, $\sin \frac{a}{2}$, $\cos \frac{a}{2}$,
AND $\tan \frac{a}{2}$.**

BY A. BABBITT,

The University of Minnesota, Minneapolis, Minn.

Let $\angle XOA = a$. Join A with X and X'. Draw $AB \perp OX$, $CD \perp OX$, and $OC \perp AX$.

$$\angle AX'X = \angle XAB = \angle AOC = \angle COX = \frac{a}{2}.$$



Also $OA = OX = X'O$.

$$\text{I. } \sin \frac{a}{2} = \pm \sqrt{\frac{1-\cos a}{2}}$$

From $\triangle OAC$,

$$\sin \frac{a}{2} = \frac{AC}{OA} = \frac{AX}{2OA} \quad (1)$$

To determine AX , we have,

$$\begin{aligned} AX^2 &= AB^2 + BX^2 = (OA^2 - OB^2) + (OX - OB)^2 \\ &= OA^2 - OB^2 + OX^2 - 2OX \cdot OB + OB^2 \\ &= 2OX^2 - 2OX \cdot OB = OX^2 \cdot 2 \left(1 - \frac{OB}{OX} \right) = OA^2 \cdot 2 \left(1 - \frac{OB}{OA} \right) \end{aligned}$$

$$\text{Hence, } AX = \pm OA \cdot \sqrt{2 \left(1 - \frac{OB}{OA} \right)} = \pm OA \sqrt{2(1-\cos a)}.$$

Substituting in (1), we get,

$$\sin \frac{a}{2} = \pm \sqrt{\frac{1-\cos a}{2}}.$$

$$\text{II. } \cos \frac{a}{2} = \pm \sqrt{\frac{1+\cos a}{2}}.$$

From $\triangle OAC$,

$$\cos \frac{a}{2} = \frac{OC}{OA} = \frac{X'A}{2OA}, \quad (2)$$

$$\begin{aligned} \text{But, } X'A^2 &= AB^2 + X'B^2 = AB^2 + (X'O + OB)^2 = AB^2 + X'O^2 \\ &\quad + 2X'O \cdot OB + OB^2 \\ &= X'O^2 + OA^2 + 2X'O \cdot OB = 2OA^2 + 2OA \cdot OB = \\ &\quad OA^2 \cdot 2 \left(1 + \frac{OB}{OA}\right). \end{aligned}$$

$$\text{Hence, } X'A = \pm OA \cdot \sqrt{2 \left(1 + \frac{OB}{OA}\right)} = \pm OA \cdot \sqrt{2(1 + \cos a)}.$$

Substituting in (2), we obtain,

$$\cos \frac{a}{2} = \pm \sqrt{\frac{1+\cos a}{2}}.$$

$$\text{III. (a) } \tan \frac{a}{2} = \pm \sqrt{\frac{1-\cos a}{1+\cos a}}.$$

$$\text{From } \triangle XAB, \tan \frac{a}{2} = \frac{XB}{AB}.$$

$$\text{From } \triangle X'AB, \tan \frac{a}{2} = \frac{AB}{X'E}.$$

$$\text{Hence, } \tan^2 \frac{a}{2} = \frac{XB}{X'E} = \frac{XO - OB}{X'O + OB} = \frac{1 - \frac{OB}{XO}}{1 + \frac{OB}{XO}} = \frac{1 - \frac{OB}{OA}}{1 + \frac{OB}{OA}} = \frac{1 - \cos a}{1 + \cos a}.$$

$$\text{And, therefore, } \tan \frac{a}{2} = \pm \sqrt{\frac{1-\cos a}{1+\cos a}}.$$

$$(b) \quad \tan \frac{a}{2} = \frac{1-\cos a}{\sin a}.$$

From $\triangle XAB$,

$$\tan \frac{a}{2} = \frac{XB}{AB} = \frac{OX - OB}{AB} = \frac{1 - \frac{OB}{OX}}{\frac{AB}{OX}} = \frac{1 - \frac{OB}{OA}}{\frac{AB}{OA}} = \frac{1 - \cos a}{\sin a}$$

(c) From $\triangle X'AB$,

$$\tan \frac{a}{2} = \frac{AB}{X'B} = \frac{AB}{X'O + OB} = \frac{\frac{AB}{X'O}}{1 + \frac{OB}{X'O}} = \frac{\frac{AB}{OA}}{1 + \frac{OB}{OA}} = \frac{\sin a}{1 + \cos a}.$$

ELEMENTARY SCIENCE OR GENERAL SCIENCE?

BY E. D. HUNTINGTON,

Normal School, Kalamazoo, Mich.

The essential difference between so-called *elementary science* and *general science* is that the former would present the elements of certain specialized sciences to the child from the standpoint of the sciences, while general science would select facts and principles from the whole field of science according to the needs of the ninth-grade child, and endeavor to present this subject matter to the child by such methods as will arouse and hold his interest. *Elementary science* would subject the child through successive semesters to the elements of physics, botany, zoology, physiology and hygiene, disguising those subjects under the titles of physical environment, plants, animals, and man. If chemistry were included, this *elementary science* sequence would represent a fairly ideal college course in the sciences, which is to be telescoped and shoved down into the high school. But, no, it is already there. Are not these courses in physical environment, plants, animals, and man but our old-time friends, physics, botany, zoology, and physiology and hygiene that have already proved such failures in the high school?

In contrast, *general science* would analyze the needs of the ninth-grade child, and *select such facts and principles from any and all branches of science as will fulfill these needs*, and by grouping this subject matter about the facts of the pupil's everyday environment, endeavor to present this subject matter in such a way that the general principles of science will be associated in his mind as phenomena of his surroundings, and not as abstract definitions.

To date, the sciences have been unsuccessful in the high school, and there is an ever-growing tendency to change science courses from requirements to electives. Science now has the right of way in every field except the high and elementary schools; it is the basis of engineering, architecture, medicine; the courts base their decisions upon the testimony of science; the scientific method has entered business and commerce; and science is even now revolutionizing and unifying the various religious creeds. The newspapers and periodicals are crowded with facts and near-facts of science; the general public is intensely interested in the discoveries and problems of science.

At such a time, it would seem that science would be the most

interesting, the most popular, and the most useful of all the subjects in the high school curriculum; that it would constitute a great part of that curriculum and be required of all students. And yet in an age when our daily activities and commonest thoughts are determined by the precepts of science, we have awakened to find that the science group is a "weak sister" in the high school curriculum, and that to most students it is neither interesting nor profitable.

What is at fault? Is the failure of science due to a faulty selection and arrangement of subject matter, or to poor teaching? The *general science* advocates hold that the failure is due primarily to the former, and that the subject matter itself has necessarily led to the poor teaching.

There has been little connection between the sciences as taught in the high school and the realities of life; to the child, botany, zoology, chemistry, and physics have each appeared as a heterogeneous mass of uninteresting and questionable facts, definitions, and rules, to be memorized and held in readiness for final exams. What he has taken away from these courses has been a hearty dislike for anything that bears the names of the individual sciences, especially botany and zoology.

The failure of the sciences in the secondary schools may be traced to two things—first, the ill-advised and so-called "logical" arrangement of the subject matter; and, second, the necessarily resulting arbitrary methods of the "forced feeding" of the unwholesome diet. Teachers of science have unconsciously in the past gone on the assumption that the division of the phenomena of nature into the highly differentiated sciences offers the proper classification under which to present those phenomena to the mind of a child. They forget that what is logical to the adult mind is often illogical to the child's mind, or even to the mind of the adult novice. *Teachers have selected and arranged the subject matter of their science courses, not from the standpoint of the needs and interests of the child, but almost wholly from the standpoint of botany, zoology, physics, or chemistry.* Naturally, the subjects so arranged have had little meaning for the child. To him, "There is no sense to it."

The second charge that the methods of teaching have been at fault brings on another storm of denial. "We point with pride" to our laboratory work, forgetting, or maybe not realizing, that a large part of the work—to many students, all of it—is merely what the overtaxed country school-teacher calls "busy work,"

and as such is not only of little value, but had better not be mentioned. Or we may "point with pride" to the splendid notebooks or examination papers that our pupils write for us. But when the student falls into the hands of an inquisitive parent, how soon is the illusion dispelled, and the whole process is proved to be a matter of vicious cramming, and not a matter understanding.

The above indictments will not be found true bills against every science course or the methods of every teacher. But just to the extent that a teacher is able to inculcate the principles of science into the understanding of the pupils, to that very extent does he draw upon the child's immediate experiences for his subject matter.

It is quite generally conceded, at least amongst science teachers, that there should be some course in science in the first year of high school, and to us the vital question is *what particular science* or *what subject matter of science* shall be selected. Botany, zoology, physics, physiography, and, of late, agriculture have all been tried and, unless presented by exceptionally skillful teachers, have been found wanting. Their failure may be quite correctly attributed to the fact that none of these subjects has proved of either much benefit or interest to the majority of students taking the course. In fact, these separate sciences have such a bad reputation that very few students enroll in the science courses except under compulsion.

The ninth-grade pupil is making his first formal acquaintance with science at a time when his whole nature is crying out for general information about his daily environment. This diversity of interests and keen desire to know affords a splendid opportunity to acquaint him with the general principles and methods of science at a time when it will make the maximum impression on the individual.

What shall be our criteria for selecting the subject matter through which the child is to make his first formal acquaintance with science? Shall we select the subject matter from the standpoint of what will be of benefit and acceptable to the child at this point, or shall our selection be determined by the classification of the specialized sciences and university requirements? Shall we select and arrange our subject matter so that it will appeal to the logic of the child, or to the logic of the advanced scientist?

If we decide in favor of the child, then our criteria must be, first, the *needs* of the child, and, second, his *interests*.

THE PUPIL'S NEEDS.

About thirty per cent of all the children who enter high school fail to return at the beginning of the second year, and so get no science training other than that which they may get in the one year. Many of the other students who continue through high school will take no further science courses unless required. Obviously, the science matter presented in this first year should be such that it will prove of maximum benefit to the individual throughout his life, and not be selected as a preparation for other science courses that are to follow. The pupil needs to become acquainted with the commonplace phenomena of his daily environment and acquire what scientific training and knowledge his immature mind will permit. The subject matter selected should be such that it will function in his daily life, and will lead to an understanding of his own body and his environment. *The fundamental principles and facts of science generally, and not of some specialized branch of science, should constitute the subject matter of the science course at this point.*

The sciences that are essentially fundamental are physics and chemistry. From a scientific standpoint, we must regard every phenomenon of the universe as a manifestation of physical and chemical laws—the atmosphere, earth, stars, plants, animals, and even life itself.

General science selects the elemental principles and facts of these fundamental sciences as the basis for its science course in the ninth grade, and seeks some group of everyday phenomena as a topic, the study of which will reveal the principles of science generally, and which will make the pupil realize that these principles are a part of his constant experience. The atmosphere serves admirably as such a topic, since it not only involves the fundamental principles of chemistry and physics, and leads directly into biological sciences, but it also holds the child's interest. It directly involves the mechanics of liquids and gases, heat and aqueous phenomena, density, electricity, and sound; and through the oxygen-carbon cycle, leads into the principles of physiology of both plants and animals. Respiration, oxidation, combustion, winds and rainfall, and health itself are among the more vital of the many aspects of this large topic, and when studied as such their close relationships are manifest.

THE PUPIL'S INTERESTS.

The classification of subject matter into our present-day sepa-

rate sciences can be little understood, and even less appreciated by the immature students. Teachers generally experience difficulty in limiting the developing child's mind to one phase of the world of nature, such as botany, zoology, chemistry, or physics. The child is interested in the bearing that all the above-named subjects have on his experiences at a particular point, and can see no necessity for holding back certain knowledge of that topic until he takes another course in another science a year or two later, or never. The young child is interested in topics from all angles; he is not concerned with matter, energy, and space, with the periodic law, or with evolution as such. But he is already interested in the atmosphere, from a study of which he gains real conceptions of the fundamentals of physics, chemistry, physiography, and biology.

METHODS OF TEACHING.

Too much of our science teaching has been a mere drill on the memorization of rules and definitions as stated in the textbooks; effective teaching of science must aim to have the child acquire a real understanding of the principles involved. But so long as the subject matter presented fails to interest the child and at the same time fails to appeal to his estimation of what is worth while, the teaching process will be the memorizing process, and the pupils will continue to say, "There's no sense to it."

The principles of science have been discovered by induction, and the teaching of science in the high school lends itself splendidly to the same method. Let textbook assignment follow, and not precede, classroom discussion and experiment. With the inductive presentation, the child is led by question and experiment to discover facts and principles for himself, and they are memorized, if at all, only after they have been comprehended. The pupils are led to discover the problems, which gives the class a keen interest in the solution of them; new problems grow out of the solutions of the present problems, and the pupil's interest is held and his mind is alert to grasp the solution.

A common objection to the introduction of *general science* into the curriculum is the assertion that it is "wholly impossible to secure teachers for a subject so broad." Any teacher who knows enough chemistry and physics to efficiently teach botany, zoology, physiology, or physiography is sufficiently well prepared in those subjects to teach *general science*.

The advocates of *general science* hold that an appreciation of the general facts and principles of science is of far greater

value to the individual, as represented by the average pupil, than is a detailed training in the facts and principles of one or two specialized sciences, and that to arbitrarily pick out those particular experiences that pertain only to a particular, specialized branch of science, while absolutely necessary for the advancement of science itself, it is out of place in the child's first and often only contact with science. *Research in science and the teaching of science in the ninth grade have very little relation, and the classification of subject matter that leads to progress in the one field leads to stagnation in the other.*

General science advocates would organize the sciences in the high school along the same lines that the courses in a particular science are organized in a university. When the university student begins his study of botany, his first course is not in morphology, cytology, ecology, taxonomy, or plant pathology. No, he must first take *Botany I*. *General botany*, if you please, must first be studied before he may attempt its specialized branches. Let us organize our high school science courses along these same lines; let the first course be an introductory course in *general science*. *Science I*, if you please, should be studied before the high school pupil attempts excursions into its specialized branches.

STUDY OF EARLY FOSSILS.

The fossil shells of the early invertebrates are of great importance to geologists, for they indicate the geologic period in which the rock beds containing them were formed—in other words, the age of the rock. Each fossiliferous rock bed contains characteristic forms or groups of forms that determine the period in which it was mud or sand. Former Director Powell of the United States Geological Survey once tersely explained to a Congressional committee the value of paleontology by saying that it is "the geologist's clock," by which he tells the time in the world's history when any rock bed was formed.

The economic importance of paleontology has been repeatedly shown in this country. In the earlier exploitation of anthracite coal, thousands of dollars were fruitlessly expended in New York in search of coal beds until the New York geologists showed that the beds in that state could contain no coal. The fossils in the New York rocks exploited are of Devonian age, whereas the fossils of the Pennsylvanian anthracite coal beds belong to the Carboniferous, a much later period. This discovery at once stopped a useless expenditure of money.

In times of doubt and perplexity, the geologist therefore turns to the paleontologist for light on the age and original order of the rock beds he is studying. The study of the animal and plant remains that are embedded in the rocks has thus become an important part of geologic work, and although the specialists who are engaged in this study are few, their work is of high importance.

SHORT STORIES OF GREAT INVENTIONS.¹

By A. L. JORDAN,

Polytechnic High School, San Francisco, Cal.

Did the Wright brothers discover aerial navigation? Marconi, wireless telegraphy? Otto, the modern gas engine? Edison, moving pictures? Morse, the telegraph? Davenport, the direct current motor? Tesla, the polyphase motor? Brush, the arc light? Faraday, current induction? Watt, the steam engine? and De Laval, the steam turbine?

Or, reversing the question, who discovered the tungsten lamp, the impulse type of water wheel, the electric furnace, the telephone transmitter, the Wheatstone bridge, the "D'Arsonval" galvanometer, electromagnetism, the reversibility of dynamo and motor, the principle of self-exciting and of compounding dynamos?

Professor Thurston says "great inventions are never, and great discoveries are seldom, the work of any one mind." The following discussion bears out that statement; but we can point out a few discoveries so great as to stand almost by themselves, as Galileo's laws of falling bodies, Newton's law of gravitation, Volta's electric battery, Edison's phonograph, Faraday and Henry's principles underlying the dynamo, Hertz's electric waves, and Roentgen's X-rays.

1. AERIAL NAVIGATION began in 1783 when a Frenchman named Rozier ascended in one of the Montgolfier brothers' hot-air balloons. Ascensions were made in America as well as in France, in hydrogen balloons, in the same year. Birdlike types of flying machines had been designed by the great Italian Leonardo da Vinci about 1,500, but not until 1852 was a soaring type successful. This machine, built to imitate an albatross, was towed by horses, and the inventor was a French sea captain named Le Bris. In the same year, Henri Giffard sent up the first dirigible. Maxim's short flight, in England, with a steam-driven machine, was in 1894; and the wonderful performances of the small machine of our own S. P. Langley were in 1897. (The work stopped most unfortunately on account of the inventor's ill-health and discouragement.) He discovered the fact that a

¹[Note: The writer acknowledges especial indebtedness to Cajori, *History of Physics* (Macmillan); Duff, *A Textbook of Physics* (Blakiston); Thurston, *History of the Steam Engine* (Int. Sci. Series, Appleton's); Talbot, *Moving Pictures* (Lippincott); *the Growth of the Steam Engine* (Int. Sci. Series, Appleton's); Talbot, *Moving Pictures* (Lippincott); and Norris, *Introd. to Electrical Engineering* (Wiley). Corrections, giving authority, will be appreciated.]

properly disposed body requires less force when the speed is great than when it is small. Lilienthal, in Germany, attracted attention to the art of gliding, and it was the news of his tragic death in 1896 which started the experiments of the Wright brothers. This was not a new interest with them, however. Years before, their father, a clergyman, had brought home a toy "helicopbre" (twisted rubbers for propulsion), and according to one of the brothers "its memory was abiding." Their first flight was in 1903, the first before witnesses in 1905, and they were closely followed by Santos-Dumont (1906), Bleriot (May, 1907), and Farman (October, 1907).

2. THE GAS ENGINE was foreshadowed by Huygens, who proposed a gunpowder engine in 1680. Papin (1690) constructed one on this plan in an attempt to lift water, and it is stated that a Professor Farish operated a small gunpowder engine about 1820. In 1838 Barnett (also in England) took out a patent for a "gas motor," and in about 1860 Lenoir (Fr.) tried a gas engine having a water jacket and electric ignition. Others, as Barsanti & Matteucci, Beau de Rochas, and Hugon made attempts, the last-mentioned using flame ignition; but in 1867 the experiments of Otto & Langen made possible the practical engine of Otto in 1876. Van Dusen of Cincinnati is said to be the first who used gasoline. Great practical work on automobile engines was done by Daimler, beginning in 1884. Others followed, among them being Panhard and Levassor in 1895.

The wonderful development since then, especially in engines for motor boats and aeroplanes, as well as for automobiles, is familiar to everyone.

3. THE DIESEL ENGINE. The working model of Dr. Rudolph Diesel's engine exploded at its first trial; but the inventor, who had thoughtfully taken refuge behind an obstacle, considered the trial a great success! From this start, made in a German workshop about 1893, has grown the modern industrial giant, used so much for stationary and marine service, particularly in Europe. We read of vessels able to travel completely around the world without addition to the fuel tanks, and, recently, of a submarine freighter crossing the Atlantic, all using Diesel engines.

The explosion in the ordinary gas engine takes place so quickly it is referred to as "at constant volume;" the combustion in the Diesel engine continues during the entire admission of the charge, the latter occurring at such a rate that the piston is under nearly "constant pressure." Another great distinction is that the

Diesel uses (in a finely atomized condition) the heavier petroleum fuels and by-product oils and tars.

The slow adoption of the engine in this country as contrasted with its adoption in Europe is explained by the fact that the first cost is high because of the necessary strength (high compression used), and because of the large number of parts; secondly, a higher class of attendance is required.

These engines are built for both two-cycle and four-cycle operation. They show by brake test the highest known thermal efficiencies, and under the best conditions combustion is so perfect that the exhaust is odorless and smokeless.

Among the engines occupying the middle ground between the Diesel and the ordinary gas engine, mention should be made of the forerunner of the Diesel, that by George B. Brayton (1872). It was not only the first constant pressure engine, but also the first gas engine in the United States. Another modern type is described as a "semi-Diesel" engine, uses the Diesel cycle and the same fuel, but starts with gasoline instead of compressed air. It gives a lower thermal efficiency, but its first cost and attendance costs are less.

4. MOVING PICTURES. The principle of "persistence of vision," upon which moving pictures depend, was enunciated by Leonardo da Vinci, but the toy known as the thaumatrope or zoetrope (slotted cylinder, figures showing a few changes in posture) has been known for centuries. One form of the apparatus, called the "phenakistoscope," was invented by a Dr. Roget and improved by Plateau in 1829. The thaumatrope was brought to public attention by Sir John Herschel in 1834. The next link in the chain of development was furnished by Marey, who built an apparatus in France in 1867, which actually showed pictures in motion. It had a continuous band of transparent material, but the figures were painted by an artist, instantaneous photography not yet being discovered. After that discovery, the Anglo-American Muybridge (1872) took successive snap-shot pictures of one of Senator Stanford's horses, using several cameras. A large number of pictures on one film was not possible until Eastman (of "Kodak" renown), working from 1884 to 1889, produced the first long celluloid film. Edison's busy brain had already been at work, and he had, in fact, a picture machine ready and waiting for the film. His "kinetoscope" was the result. It first attracted attention at the Chicago World's Fair in 1893. Edison did not trouble to take out English or French patents, and it is the Eng-

lishman, Robert Paul, to whom credit must be given for the first projection on a screen of modern moving pictures. This was early in 1895. It should be noted that the important feature was the stopping of the film, opening and closing the shutter, and jerking the film ahead for the next picture. Later in the same year, the French firm of Lumiere & Sons developed a similar apparatus; and one of their machines shown at the "Eden Musee" in 1896, together with Edison's "vitascopic," brought out in the same year, started the "movies" in the United States.

5. THE GREATEST DISCOVERY OF ALL is almost unanimously conceded to be that of the steam engine, "the greatest physical agent in the progress of civilization."

Hero's engine (150 B. C.) was a form of reaction turbine; Gerbert, a professor in the schools of Rheims (1125), had an "organ worked by heated water;" Leonardo da Vinci (about 1500) proposed a steam gun; Cardan (about 1550) devised a machine using the vacuum from condensed steam; Porta (Naples, 1601) raised water by steam pressure, also using the vacuum idea; and deCaus (France, about 1600) forced water up by a "steam fountain."

A different type of machine (corresponding to the impulse turbine) was the steam "hurdy-gurdy" of Branca, in Italy, in 1629. All of the foregoing types may be said to be experimental or preparatory to the period which followed.

The first machine which did actual work was that of Edward Somerset, Marquis of Worcester, in about 1663. We next hear of an alcohol engine, using a surface condenser, proposed by Hautefeuille (1678). This appears to have been forgotten. In 1690 the Frenchman Papin devised the first engine with a piston, and in 1698 Thomas Savery patented an apparatus for lifting water from the Cornish mines. Two vessels were filled with steam alternately, the maximum lift being 24 feet. This type is still in use; it is known as the "pulsometer." It may be of interest to note that the story of the "boy and the teakettle" is told of Worcester, of Savery, and of Watt. Thomas Newcomen in 1705 produced a machine where a jet of water was thrown into the cylinder. In all these engines, the valves were worked by hand. Now we come to the story of Humphrey Potter, the boy who in 1713 devised a self-acting valve motion which he called a "scrogger." With practically no other improvement, the steam engine remained as it was for about forty years.

James Watt in 1763 was repairing a model of an engine (New-

comen type) from a collection in a college in Glasgow. In 1765 the idea of the separate condenser struck him, and he then began his great work. Though sometimes discouraged and reduced to poverty, forced to discontinue his experiments and go to work at surveying for support, he persevered; and the next ten years saw the addition of the governor, expansive working, the indicator, the principle of compounding, etc. He found the steam engine a crude and imperfect apparatus, and left it a marvelous instrument for the progress of the world.

(To be continued.)

VASTNESS OF GRAND CANYON.

Few persons can realize on a first view of the Grand Canyon that it is more than a mile deep and from eight to ten miles wide. The cliffs descending to its depths form a succession of huge steps, each three hundred to five hundred feet high, with steep rocky slopes between. The cliffs are the edges of hard beds of limestone or sandstone; the intervening slopes mark the outcrops of softer beds. This series of beds is more than 3,600 feet thick, and the beds lie nearly horizontal. Far down in the canyon is a broad shelf caused by the hard sandstone at the base of this series, deeply trenched by a narrow inner canyon cut a thousand feet or more into the underlying "granite." The rocks vary in color from white and buff to red and pale green. They present a marvelous variety of picturesque forms, mostly on a titanic scale, fashioned mainly by erosion by running water, the agent which has excavated the canyon.—U. S. Geological Survey.

CLASSROOM SAYINGS.

Here is an answer from a Japanese boy, to the question: "Explain, with diagram, how Roemer discovered the velocity of light." He has a satisfactory diagram, but this is his explanation: "Roemer discovered at E' completed one planet in 42.5 hr. but the next planet as the earth moving farther from Jupiter it occur 1000 sec. later. Therefore he found that the light pass the diameter in 1000 mi. per sec. but later found out the speed of light by the aid of instruments in this country it was 889+mi. per sec."

A native-born American, in explaining how the power at Niagara Falls operates the street cars of Buffalo, said: "The falling water turns the turbans, and this runs the dynamite, which makes the electricity for the street cars." (This pupil had not yet studied electricity.)

Here is one more, which I give because it came from one of my pupils, and is funny, though it does not pertain to science. In preparing my room for Memorial Day, I asked, "What do we call May 30th?" "Decoration Day," said one. "And what does that mean?" "The day they signed the Decoration of Independence," answered a very bright fifth-grade boy.

RESEARCH IN PHYSICS.

Conducted by Homer L. Dodge,

State University of Iowa, Representing the American Physical Society.

It is the object of this department to present to teachers of physics the results of current research. In so far as is possible, the articles and items will be nontechnical, and it is hoped that they will not only help the teacher to keep in touch with the progress of the science, but also furnish material that will be of value in the classroom. Suggestions and contributions should be sent to H. L. Dodge, Department of Physics, State University of Iowa, Iowa City Iowa.

THE NEW WORLD OF THE ELECTRON.

By L. P. SIEG, PH. D.,

Associate Professor of Physics, State University of Iowa.

In these modern days, the discovery and describing of a new world, or even a new portion of our own world, is a feat of no little moment. The astronomers have mapped the heavens, and the millions of stars are in their places. The geographers have penetrated nearly every corner of the earth, and infrequent enough is the announcement of new lands. In spite of this infinitude of domains, both in space and with us on earth, our dreamers have added yet other places. Arcadia, the real, was turned to an Arcadia listening to the pipes of Pan. Atlantis was set by Plato in the mysterious heart of the restless Atlantic. Virgil lead Dante, trembling and fearful, through the gloomy gates of Dis. And passing through he read, and shuddered as he read, "Lasciate ogni speranza, voi ch' entrate." All these possessions, all these worlds are ours, be they real, or be they only the tenuous dreams of godlike men.

Tonight, it is my privilege to act in my humble way the role of Virgil, and to lead you through the portals of a new world. But let there be written over these gates, "Avete ogni speranza, voi ch' entrate," for we who enter here will have, nay, must have, hope. This new world is not altogether a real world, neither is it a world wholly of the imagination. It has elements of reality, and perhaps more elements of fancy. You have all seen maps drawn by the ancients. The discovery of a few landmarks sufficed for the cartographer—the remainder he filled in from fertile imagination. So you will find it with this new world that

I have called the world of the electron. This is a little world, an infinitesimal world, so small indeed that although it has existed since the remote dawn of time, it has only in recent years delivered up its age-old secrets to men. Smallness, however, must be no bar to our explorations, or else I shall be but an indifferent guide. I need not emphasize, surely, that mere smallness is of no consequence. In fact, time and space are only relative. What from one standpoint is large, is from another an infinitesimal thing. What from one viewpoint is an age, from another is but the duration of a heartbeat.

So, with this brief preamble, let us to our task. We are setting out on an adventure, and imagination is our craft. This is to be no idle May-day voyage. Each one aboard must work his passage with mind alert. It would never do at all for your guide to say, "This is so," or "That must be." At the risk of mutiny, I must try to outline to you, at least the most important steps by which we have gained knowledge of this new world. Otherwise the world will be only a world of fancy after all, to be shaken off afterwards as one throws off a grotesque dream. This makes, then, a severe demand on your attention, but the result, let us trust, will be worth the effort.

Let us start with a very real and a very solid object—say, a rod of brass. We examine it closely, but are met with the solid, cold, smooth walls of its exterior. Bring in a microscope. We must see what this thing really is, that we call brass. Put on the highest power lenses, and, behold, your brass no longer possesses an even, cold exterior. Wonderful crystals meet the view, intricate in pattern and diverse in form. This is excellent, but cannot we go further? Is the brass always solid? Is it always like this, regardless of the amount by which we magnify it? We have long ago discovered that the most powerful microscope is incapable of revealing to us the finest structure of which matter is capable. No matter how high the power of our instrument, we still see in its field, in the example we have chosen, a shining mass of crystals, apparently forming a continuous solid. Let us now pause in our efforts of discovery of the minute structure of the material of our rod by such direct means as the microscope, and rather attempt to draw information from an indirect attack. We know, for example, that when the brass rod is heated, it increases in length. Also, it is known, at least to physicists, that one can, by exerting enormous pressure, force water through solid brass. Both these facts point to a certain

discontinuity in the structure of the brass. No continuous structure could expand, nor could it permit the passage of water through its substance. Even the fact that we can bend and stretch such a substance as brass is sufficient evidence of the inherent discontinuity of its structure.

Now that we are started on our voyage of discovery of the new world, it will not be necessary to cling to our first illustration. We are at present in the business of trying to bring evidence of the discontinuity of matter. Take a liquid—say, water. Water has not to be left long in an open vessel before it has entirely vanished from sight. We say that it has evaporated. The other state of matter formed as a result of the evaporation, we call a gas, or vapor. In neither the liquid nor the vapor state can you, with any logic, assume matter to be continuous. If, not to multiply illustrations, we are convinced that matter is discontinuous, then we are forced at once to the query as to the nature of the ultimate constituents of matter. By their nature, we mean to include all their properties, their appearance, structure, and states of motion. It is possible, by the use of proper chemicals, to fill a glass of water with a myriad of small, floating particles—an emulsion, we call it—appearing as white as milk. If we focus a microscope on a few of these particles—for not all of these are below the limits of the power of the microscope—we find that some very strange motions are taking place. One seeing it for the first time is strangely reminded of the dancing of motes in the sunlight, for before our eyes is a veritable *Danse Macabre*. This simple experiment teaches us an additional fact. Not only is matter—this liquid, for instance—made up of separated parts, but these parts are in continuous motion. This peculiar motion in the case of fine, suspended particles is called the Brownian Movement, and is one of our most useful phenomena in enabling us to gather information concerning the smaller constituents of matter. The dancing of these small particles is not, of course, the actual motion of the ultimate particles of the water. However, their irregular motions are caused by collisions with much smaller, invisible, and more rapidly moving particles. Those who have read elementary physics will remember that this unceasing dance of these atoms of matter is the phenomenon that constitutes the heat energy of matter. Only when a body has lost all its heat and is at the absolute zero of temperature, 460^0 F. below the freezing point, do these atoms come to rest. One further simple illustration, taken from our common experience.

This time, let us consider a gas. We are already agreed that this gas is made up of discrete particles. Suppose one contemplates a child's toy balloon. The thin rubber membrane is obviously stretched nearly to its limit. What is stretching it? How can a number of distinct particles, separated by rather large distances, cause the bag to be so distended? One might rather suppose that the elasticity on the rubber would compress the individual particles to a smaller volume, and to a still smaller volume. We can account for the facts in a simple manner by assuming that these small particles that make up the gas are moving hither and thither with rapid pace. When they strike one another or the side walls, they must be supposed to rebound with undiminished speed. It is, then, these repeated collisions with the walls, and the subsequent rebounds, that cause the walls to be stretched ever outward. Indeed, we can liken the gas in this toy balloon, our brass rod, or any form of matter, to a swarm of bees. The swarm keeps its shape, but the bees are moving ceaselessly. So, too, are the atoms of all matter.

We are now possessed of three capital facts: First, matter in the last analysis is made up of discrete particles; secondly, we cannot hope to see the ultimate structure of matter with the microscope; and, lastly, these separate particles are moving in irregular paths with large velocities and suffering frequent impacts. We are still not in sight of our new world, but we are journeying in the right direction. Let us next consider what is the size and mean separation of these ultimate particles. The answer to this will depend on the kind of matter we are considering. Parenthetically, as a matter of comparison, let me state that the smallest particle we can ever hope to see with a microscope, so as to be able to recognize its outline, is about one one-hundred-thousandth of an inch long. If a small sphere were less than this in diameter, we might be able to detect its presence, but we could not hope to make out its form. Now consider the smallest particle that goes to make up any gas, such as air, oxygen, or hydrogen. These particles we call atoms. In the case of our brass rod, or of any other substance that is not an element, the smallest particle we call the molecule. When the molecule is subdivided, we have atoms of the elements, but no longer are we dealing with portions of matter which can be called the same material as the compound substance we have analyzed. Now, with one of these small balls, one one-hundred-thousandth of an inch in diameter, as a unit, let us see how many of these atoms

say, of hydrogen, which are the smallest atoms in existence, could be placed inside the ball. The experiments of physicists tell us that one could conceal 80,000 of these particles within the boundary of the ball. That is the same as saying that one would have to assemble together in the form of a sphere 80,000 of these hydrogen atoms before he could determine with our most powerful microscope that he had built up a structure of the form of a sphere. Or, stating it still differently, it would take four hundred of these atoms in a row to make a length equal to the diameter of one of the small balls which, we have stated, is at the limit of the microscope, and that it would take 100,000 of these groups of 400 atoms, or forty million, side by side, to make a length of one inch. However, if we have a vessel of hydrogen gas at ordinary temperature and under ordinary pressure, the atoms would by no means be so crowded as to be side by side. In fact, under normal conditions, the spaces between atoms are probably as great as ten times the diameter of the atom. Even with this wide spacing, there are a million times a million, times four hundred fifty million atoms of hydrogen in the space of a cubic inch.

We are now approaching our new world, although we have not yet reached it. We shall see that getting our imaginations down to the atom is carrying us only to the boundary of the solar system of which our new world is a part. This atom, which is the solar system containing the new world we are to explore, has, in the case of hydrogen, we have stated, a diameter of one forty-millionth of an inch. It is hard to realize what a small number like this means. To make it clearer, suppose that we magnify everything to such an extent that a small marble is made the size of the earth. With the same magnification, our hydrogen atom would be less than two feet in its greatest dimension. When I speak of these hydrogen particles, if they are unfamiliar to you, it will be easy to think of air particles, knowing that they are of about the same size.

Now that we know their size, how fast are the atoms of matter moving about? That depends upon two things. It depends upon the temperature, and it depends upon the size of the atom that we are considering. In the case of the very small hydrogen atoms, when the temperature is that of ice, these particles are moving about with an average velocity of nearly a mile a second. Some of them will be going much faster, and some of them much more slowly, but their average velocity will be about

that given. When the temperature is lower, or when the mass of the atoms is greater, then they will move with smaller speed. With opposite conditions, the opposite will be found to be true.

You will note that up to this point, I have made no attempt to give you any picture of the actual appearance of this atom. From a philosophic standpoint I suppose it is ridiculous to speak of drawing a picture of a thing that is hopelessly and forever beyond the power of our optical instruments. Granting our inability to get an exact picture of this tiny microcosm, we, nevertheless, have been able, in our scientific laboratories, to penetrate its mysteries so far that we have a pretty good idea of the relative size and of the component parts of this structure. The world we are about to study is on the borderland of reality and fancy, but there are sufficient clean-cut landmarks to take it out of the realm of pure fancy.

In getting the proper setting for our new world, we must consider the nature of the last and finest division of another sort of physical quantity. Later, I hope to be more explicit, but, for the present, I am going to consider that there is a final division of what we call electricity, and that we have a definite concept of the least possible particle of electricity, which we call the electron. For the present, I want to call to your attention illustrations of cases where these two least subdivisions, the atom of matter and the electron of electricity, are associated. We all know something about electroplating. We know that if a brass or steel spoon is supported in a proper liquid, and attached to one terminal of a source of electric current, and if the other terminal of the source is attached to a piece of pure silver, then when the current flows there is deposited on the baser metal a coating of silver, the silver plate in the meantime diminishing in size. It is when we stop to consider how this silver gets over to the spoon from the plate that we come across just the illustration we need. The silver is torn off from the plate, atom by atom, and each atom travels across to the spoon, carrying with it what we call a positive electric charge. Close examination shows us that the amount of electricity in this positive electric charge is exactly the same as that constituting the electron, but it is just the opposite in its sign. This is explained by assuming that the neutral silver atom has had an electron forcefully torn away from its structure, thus leaving the atom charged positively. This is very similar to a law in algebra. If you take a negative quantity away from zero, there

remains a positive quantity. So if you remove a negative charge from a body having a zero charge, you leave a positive charge. This very common experiment tells us an extremely important fact. We gain from it that the atom is, after all, not the least division of matter, for, behold, the atom itself is capable of losing a thing that we call the electron. Of course, if the atom is capable of losing a part of itself, it can no longer be the least conceivable division of matter. Probably more important still is the discovery that matter and electricity are most intimately connected, and, indeed, I hope within the next few minutes to be able to show you that the entity we call matter and the entity we call electricity are one and the same thing. It might be mentioned, in connection with the illustration of electroplating, that there are times when the atom, instead of losing one of these elementary charges, loses two, or even more; and again there are times when it gains one, two, or more of these. Note carefully, however, that we have never found a case where an atom has lost or gained a fractional portion of one of these unit charges. Of course, if a fractional portion of one of these elementary charges could exist, then the thing we have called a unit would no longer be considered as a unit. A real elementary portion of a substance is not divisible. We are now right down into the new world we are to think about for a little time longer. The atom, our infinitesimal solar system, has been found to possess at least one planet, the electron, and perhaps we may find that the electron occupies the same position in its solar system, that our earth finds in our own gigantic solar system. I still have not attempted to give you any idea of the exact appearance of this new system. I have had to refrain even from indicating how we know about all these rather startling statements that I have been attempting to place before you. Let it suffice for me to assure you that we know practically as much about the relative sizes and numbers of these atoms and electrons, as we know about similar quantities in our own solar system with its attendant planets.

Now that I have mentioned the electron, let me tell you of the experiments that we physicists can perform to set this elemental charge of electricity free from matter, so that we can measure it and learn of its properties. If we attach the two terminals of our electric circuit to two wires that run into a glass bulb, removing nearly every trace of air, and then if we send through the evacuated bulb an electric discharge at very much higher pressure than was the discharge through the electroplating bath,

we note some very curious phenomena. I wish it had been possible to bring some of our laboratory apparatus with me so that I could show you some of these things that I request you to take on faith. From the same electric terminal from which we previously had supposed the spoon to be supported, there is seen shooting out straight across the tube a luminous stream. Careful experiments have shown just what this stream is. It is really an army of electrons, not attached to atoms now, crossing the tube with well-nigh incredible speed. Let me give you some idea of the speed, mass, and size of one of these electrons. How we know these things, I must forbear to state, for time would not permit even the beginning of an explanation. These particles of electricity rush across the tube with speeds that depend on the excellence of the vacuum formed, but it is safe to say that a speed of 20,000 miles per second is not at all uncommon. Some go even faster than this. It might be thought that it would be a rather dangerous thing to bombard the glass wall of the tube with particles going so fast as this, if the particles have mass like ordinary matter. As a matter of fact, although the tube doesn't break, it does get very hot, and as these particles do seem to have all the properties of mass, when they strike, they not only develop a high temperature in the tube, but they originate waves of electric disturbance that all of us know by the name of X-rays. The mass of this electron is indeed very small, for I must tell you that it appears to be only one seventeen-hundredth of that of the atom of hydrogen. This latter atom, I told you, is the smallest thing that we know of that is still what we call matter. The mass of the electron is found by comparing how difficult it is to deflect it from its straight line flight with the difficulty of deflecting the atom of hydrogen. We cannot hope to weigh it with any scales we could find, for they would not be sensitive enough; moreover, the electron would not be still. And *mirabile dictu*, if we should succeed in making this electron move with a much larger velocity than 20,000 miles per second, we should find that it would possess a greater mass than one seventeen-hundredth of the mass of the hydrogen atom. In fact, if we could get this small particle to move fast enough, we should find to our amazement that its mass would get greater and greater, and, if we could get it to go with a speed of 186,000 miles per second, its mass would be greater than that of the earth; it would be infinite. In view of this fact, our new world, in spite of its diminutive size, is gaining some importance in

the universe. Now, how large is this mysterious stranger? You may have thought the hydrogen atom was a small thing, and I know I had a hard enough time in thinking of some simple comparison to make its smallness realizable. My task here is even greater. If we should magnify a hydrogen atom and an electron until the former is as large as our earth, the electron would be only eight feet in its greatest dimension. The electron is as much smaller than the atom as the atom is smaller than the smallest thing we can see with our most powerful microscopes. We must not confuse size and mass. While as far as mass, or inertia, is concerned, it would take 1,700 electrons to equal a hydrogen atom; as far as volume is concerned, you could place 1,000,000 times 1,000,000 times 200,000,000 of these electrons inside a hydrogen atom. Or you could place six million of them side by side, and the length attained would be equal only to the diameter of the hydrogen atom. You know, of course, that the most remote planet in our solar system is Neptune, nearly three billion miles from the sun; so far away, indeed, that if there were inhabitants on Neptune, they would have to wait 165 of our years from summer to summer. Let the hydrogen atom be magnified until it forms a huge ball just the size of Neptune's orbit, about six billion miles in diameter. With the same magnification, the electron would be a small satellite, one thousand miles across. Here, then, is a veritable miniature solar system, and the electron is one of the planets of the system. I still have not told you how the atom looks. For all I know, you may have the idea that the removal of the electron from the atom is like taking a tiny nick from the surface of a hard, smooth ball. If you do think this, I hope very shortly to correct the picture.

Before I do so, however, it is pertinent to inquire if there are any other phenomena in physics where we can get evidence of the appearance of the electron. Yes, they are by no means uncommon. The glowing tungsten filaments of your incandescent lamps are sending a perfect swarm of electrons to the side walls of the glass bulb. The light from the sun that bathes the earth is filled with a myriad of electrons, and at times, when these get tangled in the magnetic field of the earth, they stream down toward the magnetic poles in the beautiful flickering banners of the northern lights. Some substances, a very few of them, send off these electrons spontaneously, heating not being necessary. The element uranium is an example of this, and it was from this element that the wonderful element, radium, was discovered, and

extracted. Radium must occupy our attention for a very brief time, for we need some of the facts from it, in order to complete our picture of the atom.

The careful examination of the action of radium has revealed that it not only sends off a steady shower of these electrons, but it also sends out rays similar to X-rays, just as we might expect. In addition to these two radiations, there is sent out another kind of electrified material that scientists have called the alpha particle. This particle is electrical, but it is not at all the same as the electron. It has a positive charge, and it seems to be much heavier. Close study has shown that its charge is positive and in quantity equal to two units. If we add two electrons to it in an attempt to neutralize its charge, lo! we have built up an atom. This atom is of a gas just a little heavier than hydrogen. It is helium. The alpha particle is the thing we have been seeking to complete our picture of the atom. It is the other part, which, added in this case to two electrons, makes up a helium atom. The alpha particle is four times as heavy as the hydrogen atom, and so has a mass of four times 1,700, or about 7,000 times that of the electron. We should expect it then to be very large, and, until quite recently, we thought it must be extremely large compared to the electron. As I said a few minutes ago, the hydrogen atom is as much larger than the electron as the earth is larger than a sphere eight feet in diameter. Very recent experiments however, have shown that the alpha particle, which, remember, is a helium atom minus two electrons, and which is about 7,000 times as heavy as the electron, is, after all, vastly smaller in its dimensions than even the electron. Here is indeed food for profound thought. The electron is not the smallest thing in existence. This alpha particle, though of vastly greater mass, is probably about 500 times smaller than the electron. What of our atom now? What of the helium atom, to be specific? What is its shape? Take away two electrons and the alpha particle, and you have taken away the whole atom of helium, and yet the electrons and the alpha particle put together would not fill up an infinitesimal corner of the sphere that we have tentatively pictured as the atom. The atom, then, is not a ball, and when we state that its diameter is so and so, we must continue by stating that by diameter we mean the diameter of the orbit of the electron that happens to be furthest from the center of the atom. The center of the helium atom, for example, is this same alpha particle. Recent analysis has shown that the alpha particle is a mass of condensed

electricity having four positive units and two electrons in its make-up. The nucleus or center of the hydrogen atom is probably one positive unit 2,000 times as condensed in the space it occupies as the electron. It is the smallest thing in the universe. Let me repeat here, for the sake of greater clearness, the relative sizes of these small things, the smallest two of which are the essential constituents of what we call the atom. The least sphere we can see with a microscope is about one one-hundred thousandth of an inch in diameter. It would take four hundred atoms of hydrogen gas, side by side, to make a distance equal to the diameter of this sphere. It would take 6,000,000 electrons, side by side, to make a distance equal to the diameter of the atom of hydrogen. Lastly, it would take the cube root of 2,000, or about fourteen of the positive nuclei of the hydrogen atom, side by side, to make a distance equal to the electron's diameter. This positively charged nucleus is the sun of this new electron world system, and the electrons are the whirling planets, passing round and round their central sun, marking surely the passage of the days and the seasons, and the years. This central sun is much more concentrated than is our own sun, and it is all electricity. The electrons are all electricity. Take these away, and there is no longer an atom. Gold differs from lead, only in the relative composition of nucleus and outer whirling electrons. Matter, then, is electricity, and they are one and the same.

Come, then, let us finish our voyage. Come in thought until we are resting together on the surface of this new world, as it is found in the solar system, helium. We have seen some happy fairy, and with her wand she has touched us. Our poor human eyes can see little of what is going on in this strange place, and our ears hear no sound. There is no sensation of heat or of cold, for those are grosser things of another and far-off world. There is light here which comes from the whirling electrons, and let us hope that we see the distant sun (the alpha particle), just as in that other world which we left but now, we used to see our own sun. There is no burning disc to this sun—just a point of light. Far off through space we see our companion planet, the other electron of this helium atom, as it moves majestically through the night.

Matter and electricity, we muse, are one. All matter is made up of these two kinds of electricity—the electron and the very highly condensed positive center. Atoms, then, are base pretenders when it comes to occupying space in the universe. Most

of the atom is betweenness, may I say. You could get inside the atom with a lantern and look with the persistence of a Diogenes, seeking for ages, and fail to find anything there. The nearest picture I can call up for something with which to compare our rod of brass is our starry universe round about us. If we could be made as small as electrons, we could wander an age within the boundaries of the brazen rod, and never find aught but space.

I want to make this picture of the atom clear to you, even at the cost of repetition. Suppose an atom of silver, for example, is magnified until it is the size of this room, and that we are stationed in a convenient post for observation. Our first casual glance will reveal nothing. Looking more closely, we should see at the center of the room a small object not more than one twenty-fifth of an inch in diameter, and probably less. Closer inspection of this object would show that there are about fifty electrons and about one hundred positive units of electricity all crowded together. Casting our eyes about the room again, we should note that there are some fifty more electrons whirling about this central ball, some of them near it, and some as far away as the bounding walls of the room. But for all these objects, the room is practically empty. As I said, most of the atom is betweenness.

Dreamers, all of us, and the most incorrigible of all are these students of physics. A topsy-turvy world indeed with the scientist outdoing the poet in fancy. Surely, surely, fact is stranger than any fiction.

And now we must come back from this believing voyage, leaving reluctantly enough our new-found possessions. Some day we shall return thither, and explore these planets and their central suns. Mayhap then, another portal will be found yawning wide, and over the shining entrance will be a messenger of God, pointing to gleaming letters of fire. And these words are there writ large and clear: "Pass on through, beloved children of men, it is right that ye should seek. Toil on, but know that ye will find neither beginning nor ending except it be in me."

Announcements

School Science and Mathematics has been the pioneer in putting into operation and fostering those branches of Science and Mathematics teaching, which have been most helpful to the progressive teacher. It has catered and is catering to all departments of Science and Mathematics teaching below the college and university grade. To better further these endeavors prominent teachers and special writers are being invited to take charge of particular departments. The appointment of Professor H. L. Dodge of the University of Iowa to take the management of the Research in Physics has been extremely fortunate to everybody concerned.

We are pleased to announce that Professor Frederic D. Barber of the State Normal University of Normal, Illinois, and author of a well received text in general science, has consented to act as director of our General Science department. The editorial duties of this particular phase of science work has heretofore been distributed over our other science departments. Under Professor Barber's direction general science teachers will find in this journal articles of information which will be of the greatest assistance in their line of instruction. Teachers are asked to transmit to the editor, articles and helpful notes on this comparatively new phase of science instruction.

In our February issue we are proposing to open a department of mathematics questions. This department will be in charge of our Mathematics editor, Professor Herbert E. Cobb of Lewis Institute, Chicago. Teachers of mathematics are asked and expected to transmit to the editor questions and problems involving the pedagogy of mathematics teaching, which they would like to have clarified.

We have in contemplation the formation of other departments. Suggestions for the betterment of Science and Mathematics instruction are always gratefully received.

PROBLEM DEPARTMENT.

By J. O. HASSSLER,
Englewood High School, Chicago.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics. Besides those that are interesting per se some are practical, some are useful to teachers in class work, and there are occasionally some whose solutions introduce modern mathematical theories and, we hope, encourage further investigation in these directions. All readers are invited to propose problems and solve problems here proposed. Problems and solutions will be credited to their authors. In selecting solutions for publication we consider accuracy, completeness, and brevity as essential. Address all communications to J. O. Hassler, 2301 W. 110th Place, Chicago.

Algebra.

486. Proposed by N. P. Pandya, Sojitra, Dt. Petlad, India.

In a division, the dividend consists of five digits and the quotient exceeds the divisor by 16; the divisor exceeds the remainder by 55. Find (i) the least possible dividend; (ii) the value of the digit d if the dividend is 58d79.

I. Solution by Norman Anning, Chilliwack, B. C.

If d is the divisor, then the dividend is

$$d(d+16)+(d-55) = d^2+17d-55.$$

Since the dividend has 5 digits, we have

$$9,999 < d^2+17d-55 < 100,000,$$

$$10,054 < d^2+17d < 100,055,$$

$$92 < d < 308.$$

(i) The least possible divisor is 93 and the least possible dividend is $93 \times 109 + 38 = 10,175$.

(ii) The dividend lies between 58,078 and 58,980,
 $58,133 < d^2+17d < 59,035$,

$$232 < d < 235.$$

Trying 233 and 234, we find that

$$58,679 = 234 \times 250 + (234-55).$$

The required digit is 6.

II. Solution by Everett T. Owen, Oak Park (Ill.) High School, and James H. Weaver, West Chester, Pa.

Let x = the divisor.

Then $x+16$ = the quotient and

$x-55$ = the remainder.

$$x(x+16)+x-55 = 10,000+k, \quad \text{or}$$

$$x^2+17x-(10,055+k) = 0.$$

$$x = \frac{-17 \pm \sqrt{40,509+4k}}{2}.$$

Since k must be positive and x integral, and $202^2 > 40,509 > 201^2$, and the value of the radical odd, the smallest possible value of the radical is 203. This gives $k = 175$, and the number is 10,175. The divisor is 93, the quotient is 109, and the remainder is 38.

When the dividend is 58d79, forming an equation similar to (1) gives

$$x = \frac{-17 \pm \sqrt{232,825+400d}}{2}.$$

For the smallest value of the radical satisfying the conditions of the problem, $d = 6$, and the number is 58,679.

487. *Proposed by Albert Babbit, University of Minnesota, Minneapolis.*

If $c_0, c_1, c_2, \dots, c_n$ be the coefficients of the terms of the expansion $(1+x)^n$, show that $c_0 - \frac{1}{2}c_1 + \frac{1}{3}c_2 - \dots - \frac{(-1)^n}{n+1} c_n = \frac{1}{n+1}$.

(From an examination paper of the Actuarial Society of America, May, 1915).

I. *Solution by Murray J. Leventhal, Stuyvesant High School, New York City.*

In the series $a_1, a_2, a_3, \dots, a_n, a_{n+1}$,

$$a_{n+1} = a_1 + nd_1 + \frac{n(n-1)}{2!} d_2 + \dots$$

where d_1, d_2, d_3 , etc., are the first terms of the 1st, 2nd, 3rd, etc., orders of differences. When $a_1 = c_0 = 1$, $d_1 = -\frac{1}{2}$, $d_2 = \frac{1}{3}$, etc., the right member of (1) becomes $c_0 - \frac{1}{2}c_1 + \frac{1}{3}c_2 - \dots$ equal to the $(n+1)$ st term of the series of numbers, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$, which is obviously $\frac{1}{n+1}$.

II. *Everett W. Owen of Oak Park, Ill., referred us to the following solution in Charles Smith's "Treatise on Algebra."*

$$\begin{aligned} c_0 - \frac{1}{2}c_1 + \frac{1}{3}c_2 - \frac{1}{4}c_3 + \dots &= 1 - \frac{1}{2}n + \frac{1}{2} \frac{n(n-1)}{1 \cdot 2} - \frac{1}{4} \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} + \dots \\ &= \frac{1}{n+1} \left\{ n+1 - \frac{(n+1)n}{1 \cdot 2} + \frac{(n+1)n(n-1)}{1 \cdot 2 \cdot 3} - \dots \right\} \\ &= \frac{1}{n+1} - \frac{1}{n+1} \left\{ 1 - (n+1) + \frac{(n+1)n}{1 \cdot 2} - \frac{(n+1)n(n-1)}{1 \cdot 2 \cdot 3} + \dots + (-1)^{n+1} \right\} \\ &= \frac{1}{n+1} - \frac{1}{n+1} (1-1)^{n+1} = \frac{1}{n+1}. \end{aligned}$$

Geometry.

488. *Proposed by Geo. Blanchard, Portland, Ore.*

If α, β, γ be the distances from the vertices of a triangle to the points of contact with the inscribed circle, show that radius of circle has the value,

$$\left(\frac{\alpha\beta\gamma}{\alpha+\beta+\gamma} \right)^{\frac{1}{2}}$$

(From entrance examination Military Academy at Woolwich, England.)

I. *Solution by Eva Crane Farnum, Manual Arts High School, Los Angeles, Cal.*

Area = $R(\alpha+\beta+\gamma)$, one-half perimeter by altitude.

Area = $\sqrt{(\alpha+\beta+\gamma)\alpha\beta\gamma}$, triangle in terms of sides.

$R(\alpha+\beta+\gamma) = \sqrt{(\alpha+\beta+\gamma)\alpha\beta\gamma}$.

$$R = \left(\frac{\alpha\beta\gamma}{\alpha+\beta+\gamma} \right)^{\frac{1}{2}}.$$

II. *Solution by Faith Canfield, student, Northwestern University, Evanston, Ill.*

From

$$r = \frac{\sqrt{(s-a)(s-b)(s-c)}}{s},$$

where a, b, c are the three sides, respectively, of the triangle, and s is one-half the perimeter,

$$r = \sqrt{\frac{(a+\beta+\gamma-a-\beta)(a+\beta+\gamma-\beta-\gamma)(a+\beta+\gamma-\gamma-a)}{a+\beta+\gamma}}$$

$$= \sqrt{\frac{\gamma a \beta}{a+\beta+\gamma}}.$$

489. *Suggested by Jack Benson, "Somewhere in France."*

[Relayed to the Editor by Norman Anning.]

Given a triangle, to construct, in the simplest way, the line from which its area may be scaled.

I. *Jack Benson's solution.*

Suppose one side, AB , of the given triangle, ABC , is greater than 2. This is no real limitation since it can always be brought about by choice of a suitable subdivision of the unit of length.

Then describe a circle with center B and radius 2.

Draw AD tangent to this circle.

Through C draw CE parallel to AB to meet AD in E .

Then the number of linear units in AE is equal to the number of square units in ΔABC .

Proof.

$\Delta ABC = \Delta ABE$ because they are on the same base, AB , and between the same parallels.

$$\Delta ABE = \frac{1}{2}BD \times AE = 1 \times AE.$$

II. *Solution by Mabel G. Burdick, Stapleton, N. Y.*

Let b = base, h = altitude of the triangle. Construct the fourth proportional to 1, h , and $\frac{1}{2}b$. This will represent the number of units in the area. (The simplest method would be by means of squared paper, laying off the numbers on the sides of a right triangle.)

Trigonometry.

490. *Proposed by Clifford N. Mills, Brookings, S. Dak.*

If $A+B+C = 180^\circ$, show that unity is the least value of $\cot^2 A + \cot^2 B + \cot^2 C$.

I. *Solution by Norman Anning, Chilliwack, B. C.*

The expression,

$$(\cot B - \cot C)^2 + (\cot C - \cot A)^2 + (\cot A - \cot B)^2,$$

is zero or positive according as the angles are all equal or not.

Under the same conditions,

$$\cot^2 A + \cot^2 B + \cot^2 C \leq \cot B \cdot \cot C + \cot C \cdot \cot A + \cot A \cdot \cot B.$$

But when $A+B+C = 180^\circ$, the sum on the right is equal to unity. Hence the theorem.

II. *Solution by Dewitt T. Weaver, Agricultural High School, Middletown, Va.*

Let a, b , and c be the sides of the triangle, and let $s = \frac{a+b+c}{2}$.

$$\text{Let } \Delta = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$\sin A = 2 \Delta / bc, \quad \sin B = 2 \Delta / ac, \quad \sin C = 2 \Delta / ab.$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ac}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

$$\therefore \cot A = \frac{b^2 + c^2 - a^2}{4 \Delta}, \quad \cot B = \frac{c^2 + a^2 - b^2}{4 \Delta}, \quad \cot C = \frac{a^2 + b^2 - c^2}{4 \Delta},$$

$$\cot^2 A + \cot^2 B + \cot^2 C = \frac{(b^2 + c^2 - a^2)^2 + (c^2 + a^2 - b^2)^2 + (a^2 + b^2 - c^2)^2}{16s(s-a)(s-b)(s-c)} \\ = \frac{3a^4 + 3b^4 + 3c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2}{-a^4 - b^4 - c^4 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2}.$$

It is now necessary to prove that the numerator of this fraction is greater than or equal to the denominator for all positive values of a , b , and c .

$$a^2 + b^2 \leq 2ab, \quad a^2 + c^2 \leq 2ac, \quad b^2 + c^2 \leq 2bc, \quad \text{since } (a-b)^2 \leq 0, \quad \text{etc.}$$

$$\therefore a^4 + b^4 \leq 2a^2b^2, \quad a^4 + c^4 \leq 2a^2c^2, \quad b^4 + c^4 \leq 2b^2c^2; \quad \text{whence}$$

$$2a^4 + 2b^4 + 2c^4 \leq 2a^2b^2 + 2a^2c^2 + 2b^2c^2, \quad \text{and}$$

$$4a^4 + 4b^4 + 4c^4 \leq 4a^2b^2 + 4a^2c^2 + 4b^2c^2.$$

Subtracting $a^4 + b^4 + c^4 + 2a^2b^2 + 2a^2c^2 + 2b^2c^2$ from each member of this inequality,

$$3a^4 + 3b^4 + 3c^4 - 2a^2b^2 - 2b^2c^2 - 2a^2c^2 \leq 2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4.$$

$$\therefore \cot^2 A + \cot^2 B + \cot^2 C \leq 1.$$

CREDIT FOR SOLUTIONS.

472. A. B. Hussey.
 485. Dewitt T. Weaver.
 486. Norman Anning, Mabel G. Burdick, Everett W. Owen, Dewitt T. Weaver, James H. Weaver, R. M. Mathews. (6)
 487. Albert Babbit, C. A. Bergstresser, Murray J. Leventhal, Everett W. Owen. (4)
 488. Neil Beardsley, C. A. Bergstresser, Mabel G. Burdick, Faith Canfield, Mary W. Clark, Eva Crane Farmum, C. E. Githens, M. Helen Kelley, Murray J. Leventhal, L. E. A. Ling, Glendora Mills, R. T. MacGregor, Everett W. Owen, A. L. Schur, W. T. Short, Dewitt T. Weaver, James H. Weaver, R. M. Mathews, Emma C. Ackerman. (19)
 489. Norman Anning, Mabel G. Burdick, Everett T. Owen, James H. Weaver. (4)
 490. Norman Anning, Everett T. Owen, Dewitt T. Weaver, one incorrect solution. (4)
 39 solutions.

PROBLEMS FOR SOLUTION.

Algebra.

501. *Proposed by A. G. Montgomery, State Normal School, Athens, W. Va.*

Show that $x^{4m} + x^{2m} + 1$ never represents a prime number if x be any other integer than one.

502. *Proposed by Norman Anning, Chilliwack, B. C.*

Exhibit $(x^2 + y^2 + z^2)^3$ as the sum of three squares.

Geometry.

503. *Proposed by Clifford N. Mills, South Dakota State College, Brookings, S. Dak.*

The perpendicular from any point of a circle upon a chord is the mean proportional between the perpendiculars from the point upon the tangents drawn at the extremities of the chord. Prove.

504. *Proposed by Walker Cisler, student in West Chester, Pa., High School.*

To construct in a given circle seven equal regular hexagons, one having as its center the center of the given circle, and the other six erected on the sides of the first.

505. *Proposed by R. T. McGregor, Bangor, Cal.*

Show that the circles described on the three diagonals of a complete quadrilateral as diameters are coaxial.

ARTICLES IN CURRENT PERIODICALS.

American Forestry, for November; *Washington, D. C.*; \$3.00 per year, 25 cents a copy: "The Red Gum—Identification and Characteristics," Samuel B. Detwiler; "Commercial Uses of Red Gum" (six ill.); "Trees in Medicine," John Foote, M. D.; "Conservation of American Wild Flowers" (five ill.), R. W. Shufeldt, M. D.; "Philippine Island Timber," Arthur F. Fischer; "National Highways in Florida" (three ill.), Mrs. Kirk Munroe; "Safety First in Tree Planting" (nine ill.), Perley Spaulding and Carl Hartley.

American Mathematical Monthly, for November; 5465 *Greenwood Ave., Chicago*; \$3.00 per year: "On the Diophantine Equation $x^a + ay^a = u^2 + bv^2$," F. L. Carmichael; "Graphical Constructions for a Function of a Function and for a Function Given by a Pair of Parametric Equations," W. H. Roever; "A Note on the Sum of the Remainders of a Series," Glenn James.

American Naturalist, for December; *Garrison, N. Y.*; \$4.00 per year, 40 cents a copy: "Experimental Intersexuality and the Sex Problem," Richard Goldschmidt; "Piebald Rats and Multiple Factors," E. C. MacDowell; "Some Features of Ornamentation in the Killifishes, or Toothed Minnows," Henry W. Fowler.

Geographical Review, for November; *Broadway at 156th St., New York City*; \$5.00 per year, 50 cents a copy: "Along the Maine Coast," E. P. Morris; "The Decrease of Population along the Maine Coast," (2 maps, 3 photos) Leonard O. Packard; "The Contributions of Geodesy to Geography" (1 map, 8 photos), William Bowie; "The Passing of The Great Race" (4 insert maps in color), Madison Grant; "The Population of Florida: Regional Composition and Growth as Influenced by Soil, Climate, and Mineral Discoveries" (2 maps), Roland M. Harper; "The Distribution of People in Japan in 1913" (3 maps), Mark Jefferson.

Journal of Geography, for December; *Madison, Wis.*; \$1.00 per year, 15 cents a copy: "Tree Crops for Dry Lands," J. Russell Smith; "Economic Aspects of Inland Water Transportation" (concluded), H. G. Moulton; "The Influence of the Lumber Industry upon the Salt Industry of Michigan," C. W. Cook; "College Entrance Examination Questions in Geography," D. W. Johnson; "The Port of Kobe," Walter N. Lacy; "A Recommended List of Essentials in Place Geography," V. C. Bell, T. M. Davies, H. D. Smith.

L'Enseignement Mathématique, for September; *G. E. Stechert & Co., 151 West 25th St., New York*; 15 francs per year, 2 francs a copy: "Esquisse d'une introduction à la théorie des probabilités," E. Guillaume; "Sur une propriété des fonctions dérivées," A. Denjoy; "Extension à l'espace d'un théorème sur les cubiques planes," F. Gonseth; "Sur la représentation graphique des nombres premiers," A. Reymond; "La

préparation théorique et pratique des professeurs de mathématiques de l'enseignement secondaire en Belgique."

Nature-Study Review, for November; *Ithaca, N. Y.*; \$1.00 per year, 15 cents a copy: "A Steller's Sea-Lion that Dives," Gayne T. K. Norton; "Nature-Study and Common Forms of Animal Life," R. W. Shufeldt; "Fly Campaign in Springfield," Isadora Bennett; "Correlation Between Nature-Study and High School Biology," G. W. Hunter.

Photo-Era, for December; *Boston, Mass.*; \$1.50 per year, 15 cents a copy: "Portraiture in the Home," Norman Butler; "The Miniature Camera for Scientific Work," Lehman Wendell; "Photographic Uses for White Watercolor," William S. Davis; "Mexican Adventures of a Camera-Man," Francis A. Collins; "Dependence of Tone Upon the Character of the Negative," Dr. Theodore Körner; "Photography in Egypt," Edward Lee Harrison.

Physical Review, for November; *Ithaca, N. Y.*; \$6.00 per year, 60 cents a copy: "A Comparison of Alternating and Direct Spark Potentials," J. C. Jensen; "The Nature of the Collisions of Electrons with Gas Molecules," K. T. Compton and J. M. Benade; "Notes on the Elastic Properties of Phosphor-Bronze Wires," Arnold J. Oehler; "Temperature and Blackening Effects in Helical Tungsten Filaments," B. E. Shackelford; "The Coefficient of Viscosity of Air by the Capillary Tube Method," Effie Markwell; "The Echelette Grating," A. Trowbridge; "The Frequent Bursting of Hot Water Pipes in Household Plumbing Systems," F. C. Brown; "On the Demagnetization of Iron and Steel Rods by Strain (and Impact)," Guy C. Becknell; "A Test for X-Ray Refraction Made with Monochromatic Rays," David L. Webster and Harry Clark; "Thermionic Currents from Molybdenum," E. R. Stoekle; "Ionization by Impact in Mercury Vapor and Other Gases. I. The Direct Measurement of the Ionizing Potentials," F. S. Coucher; "The Magnetic Spectra of the β -Rays of Radium D and of Radium and Its Products, Determined by the Statistical Method," Alois F. Kovarik and L. W. McKeehan.

Popular Astronomy, for December; *Northfield, Minn.*; \$3.50 per year, 35 cents a copy: "The Problems of the Nebulae," Vincent Francis; "Chronology of the Egyptian Pyramids," F. J. B. Cordeiro; "The Motion of the Pole," O. H. Truman; "Polaris," Charles Nevers Holmes; "Report on Mars, No. 17, with Plates XL-XLV," William H. Pickering.

School World, for November; *Macmillan & Co., London, Eng.*; 7s 6d per year: "Science at the Beginning and at the End of the Curriculum," O. H. Latter; "Educational Reform and the Teacher," F. Smith; "Practical Studies," T. S. Usherwood; "Secondary Education in Australia," Prof. H. S. Carslaw; "Scientific Method in Education," S. E. Brown; "The Mapping of the Earth," Edward A. Reeves.

ERRATA.

On pages 846 and 847 of the December, 1916, issue of this Journal the credit of the Live Chemistry article should have been given to Mr. R. B. Whitmoyer of the Atlantic City High School.

Also, the number at the end of the fourteenth line from the bottom of page 846 should have been 1,200 instead of 200.

SCIENCE QUESTIONS.

BY FRANKLIN T. JONES,
University School, Cleveland, Ohio.

Readers of SCHOOL SCIENCE AND MATHEMATICS are invited to propose questions for solution—scientific or pedagogical—and to answer questions proposed by others or by themselves. Kindly address all communications to Franklin T. Jones, University School, Cleveland, Ohio.

Please send examination papers of all sorts. They will be acknowledged in these columns.

Questions and Problems for Solution.

243. *Proposed by J. P. Drake, Emporia, Kan.*

A ladder 30 ft. long and weighing 100 lbs. rests against a wall at an angle with the wall of 45° . Center of gravity of the ladder is $\frac{1}{3}$ up from the base. Coef. of friction with ground = .6; with the wall it is .3. How far up can a man who weighs 200 lbs. climb before the ladder will begin to slip?

244. *Proposed by William A. Hedrick, Central High School, Washington, D. C.*

A man weighing 124 lbs. has a spring balance weighing 8 lbs., a movable pulley weighing 2 lbs., a rope whose weight may be disregarded, and a load of 124 lbs. which he wishes to lift.

(a) He arranges the pulley for a minimum pull. He fastens the end of the rope to the *ring* of the balance, and himself pulls on the *hook*. When the load is just sustained, what will be the reading of the spring balance?

(b) He substitutes himself for the load, and again pulls on the hook of the balance. Is there any difference in the reading of the balance?

(c) He then fastens the original load to himself, and again pulls on the balance, until himself and load are just sustained. What additional pull will the spring balance indicate?

245. *From Chemical Annual, No. 55.*

50° Bé sulphuric acid contains 62.18 per cent H_2SO_4 , and 52° Bé acid contains 65.13 per cent H_2SO_4 . (a) To how many pounds of 50° Bé sulphuric acid are 350 cubic feet of 52° Bé acid equivalent? (b) If 60° Bé sulphuric acid contains 77.67 per cent H_2SO_4 , to how many pounds of 60° Bé sulphuric acid are 530 cubic feet of 52° Bé acid equivalent?

246. *Comments are requested on the following set of chemistry questions, constituting the paper set by the Board for candidates for entrance to Harvard, Princeton, and Yale in September, 1916.*

Is it too long or too short? (Time allowed, three hours.)

Is it too difficult or too easy? (Should the standard be the average of instruction in good schools, or the best instruction in the best schools?)

Is it not true that chemistry is better taught than physics?

Are the numerical problems the sort that should be given?

Please answer Nos. 247 and 248.

COMPREHENSIVE EXAMINATION IN CHEMISTRY, SEPTEMBER, 1916, 2-5 P. M.

Part I. (Answer All Questions in Part I.)

1. (a) Define the terms "molecule," "atom," and "ion."

(b) State Avogadro's hypothesis, and show how it guides the chemist in determining molecular weights.

2. (a) What takes place when steam is passed over heated zinc or iron?

(b) How would you identify the products?

(c) What do you learn about the composition of water from this experiment?

3. Describe the action, if any, and represent by equations the chemical changes taking place when dilute hydrochloric acid and nitric acid are added separately to each of the following substances: (a) ferric hydroxide, (b) zinc oxide, (c) calcium carbonate, (d) silver.

4. (a) Calculate the percentage of oxygen in crystallized copper nitrate, $\text{Cu}(\text{NO}_3)_2 \cdot 6\text{H}_2\text{O}$. (Cu = 64, N = 14, O = 16, H = 1.)

(b) What weight and volume at 0°C . and 760 mm. of carbon dioxide can be obtained by treating an excess of sodium acid carbonate with 490 gm. of sulphuric acid containing 20 per cent of H_2SO_4 ? (Na = 23, C = 12, O = 16, H = 1, S = 32.)

NOTE.—A liter of carbon dioxide at 0°C . and 760 mm. weighs 1.97 gm.

5. Define and illustrate with an example each of the following: (a) acid anhydride, (b) catalytic agent, (c) saturated solution, (d) sublimation, (e) either destructive distillation or fractional distillation.

Part II—Supplementary Requirements—Group A. (Answer Two Questions from This Group.)

6. Two rods of copper are placed in a solution of copper sulphate. What takes place when a current of electricity is passed from one rod to the other through the solution? How might you prove what happens to each of the copper rods?

7. State in words and by writing equations how you would obtain: (a) ferric chloride from ferrous chloride; (b) ferrous chloride from ferric chloride; (c) sodium carbonate from sodium hydroxide; (d) sodium hydroxide from sodium carbonate; (e) oxygen from ozone; (f) ozone from oxygen.

8. How would you prove, by chemical means, the presence in air of each of the following components: (a) water, (b) carbon dioxide, (c) oxygen, (d) nitrogen?

9. (a) Arrange the names of ten common elements in natural groups.

(b) Write the formula for one oxide of each, and state what acid or base can be formed from each of these oxides.

Group B. (Answer Two Questions from This Group.)

10. (a) A compound has the following composition: carbon, 54.67 per cent; hydrogen, 9.11 per cent; oxygen, 36.22 per cent. Find the simplest formula for the substance. (C = 12, O = 16, H = 1.)

(b) A body of air at constant pressure occupies a volume of 500 cc. at 20°C . At what temperature will its volume become 1,000 cc.?

247. How many liters of ammonia gas, measured under standard conditions, can be obtained when 20 gm. of sodium hydroxide react with an excess of ammonium sulphate? (Na = 23, O = 16, H = 1, N = 14, S = 32.)

NOTE.—A liter of ammonia gas at 0°C . and 760 mm. weighs 0.772 gm.

12. What experimental evidence can be cited to show (a) that chloride ions are not molecules of chlorine? (b) that chloride ions are charged with negative electricity?

248. Write a reversible chemical reaction, and explain how it may be made to go to completion in either direction.

Group C. (Answer One Question from This Group.)

14. (a) Make a diagram of an acetylene generator. With the aid of an equation, explain its operation.

(b) Write the equation for the complete combustion of acetylene.

(c) Why does acetylene burn with a flame which is more luminous than that of methane?

15. Answer any two of the following questions:

(a) Mention a necessary property that must be possessed by an oil

(1) for soap-making, (2) for mixing with paint, (3) for a lubricant. Name as an example some particular oil for each use.

(b) What are the physical and chemical differences between cast iron and steel?

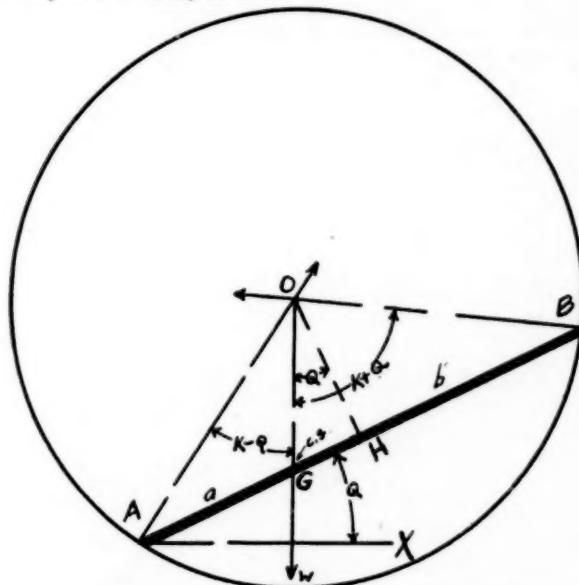
(c) Why does iron, in the course of time, turn completely into rust, while aluminum and zinc tarnish only slightly?

Solutions and Answers.

228. *Proposed by W. L. Baughman, East St. Louis, Ill.*

A rod whose center of gravity divides it into two parts a and b is placed inside a smooth sphere of such radius that the rod subtends an angle $2k$ at the center. Find the inclination of the rod to the horizon

Solution by W. L. Baughman.



Let O be the center of the sphere, G the center of gravity of the rod, Q the inclination of the rod to the horizon, and A and B the ends of the rod whose center of gravity divides it into two parts a and b , respectively.

Draw AO and BO . And OH perpendicular to the rod; also a horizontal line AX , making angle BAX acute. Then

$$\text{angle } AOG = \text{angle } K - Q,$$

$$\text{angle } BOG = \text{angle } K + Q,$$

angle $GOH = \text{angle } BAX = \text{angle } Q = \text{angle of inclination of the rod to the horizon.}$ It now follows that

$$\sin(K-Q) : \sin(K+Q) = a : b \text{ and thus}$$

$$\tan Q = \frac{b-a}{b+a} \tan K.$$

Discussion: Here it is supposed that b is greater than a . And that the c. g. of the rod lies vertically under the meeting point of the forces BO and AO , supporting the rod. In this particular case the meeting point of the forces is the center of the sphere.

To add "spice" to the above problem, I propose this question: What is the pressure on the sphere at the ends of the rod?

229. *Proposed by O. L. Brauer, Selma, Cal.*

Question: If you look at a distant light through a wire window screen, you see a symmetrical cross of light radiating from the source. If you look through the screen at an acute angle, there appears an unsymmetrical star of six (?) points. These lines of light are about twenty times as long as the diameter of the light. (The screen is made of about No. 22 iron wire, making little squares about 2 mm. on a side.) How is this phenomenon explained?

Answer by Prof. Frank P. Whitman, Adelebert College, Cleveland, Ohio.

There are probably two causes contributing to the phenomena of vision through a wire screen, which of course must be viewed optically as a pair of gratings crossed at right angles. The first cause is the common phenomenon of reflections from a succession of similar parallel objects, as when the sun is reflected in disturbed water. The succession of ripples extends the image of the sun into a long streak of light. Another illustration is the not infrequent vertical "pillar" of light seen above the sun when near the horizon, and probably due to reflections from many ice crystals, lying horizontally in the atmosphere between the sun and the observer. Two such bands, at right angles, would be formed by the meshes of the screen, and probably are the principal sources of the cross as observed by your correspondent.

A second effect of a grating is the well-known "diffraction spectrum," which gives rise to a band of colored light formed of successive spectra, extending on both sides of the central image of the source. A pair of crossed gratings gives rise to two such bands, at right angles, forming a cross. But the whole diffraction pattern is much more complicated than this. The diagonals, or rather the corners, of the squares formed by the meshes of the screen may be regarded as forming another grating, giving rise to another band of spectra, or rather to two bands, lying at forty-five degrees from the first, and forming what might be called an eight-pointed star.

A window screen is of too large a mesh—that is, the gratings are too coarse—to show these phenomena to any extent, but if the screen is inclined, as your correspondent suggests, one of the gratings becomes finer, while the other remains the same. The diffraction effects might now begin to appear, and there would result an unsymmetrical, somewhat starlike figure, but of eight points, instead of six. I can see no reason for a six-pointed star.

In J. S. Ames' *Textbook of General Physics*, page 539, there is an illustration of the eight-rayed effect due to crossed gratings. An arc light, viewed through the meshes of a silk umbrella, gives a similar, though somewhat different, diffraction pattern.

232, 233. *Also solved by A. Haven Smith, Riverside, Cal.*

236. *From a College Board Comprehensive Chemistry paper, June, 1916.*

A certain quantity of magnesium dissolved in acid gave exactly 100 cc. of dry hydrogen at a temperature of 22° C. and a pressure of 780 mm. How many grams of metal were used? Compute the result to three significant figures ($Mg = 24.3$).

NOTE.—A liter of hydrogen at 0° C. and 760 mm. weighs .09 gm.

Solution by R. W. Boreman, Parkersburg, W. Va.

Also solved by Jessie Caplin, Minneapolis, Minn.

Reducing gas to standard condition =

$$\frac{100 \times 780}{760 (1 + 0.00366 \times 22)} = 94.98 \text{ cc.} = .09498 \text{ liter.}$$

$$.09498 \times .09 = .008908 \text{ g. of H.}$$

$$24.3 \qquad \qquad \qquad 2.016$$

$$\text{Mg} + \text{H}_2\text{SO}_4 = \text{MgSO}_4 + 2 \text{ H}$$

$$x \qquad \qquad \qquad .008908$$

$$24.3 : 2.016 = x : .008908, \quad x = .107 \text{ g. of Mg.}$$

237. *From the same source as 236.*

One gram of pure iron forms 1.43 gm. of an oxide. Find (a) the percentage composition for this oxide of iron, (b) its simplest formula, and (c) the equivalent weight of iron in the compound. (Fe = 56, O = 16.)

Solution by Jessie Caplin, West High School, Minneapolis, Minn.

Also solved by R. W. Boreman.

1.43 gm. is iron oxide.

$$(a) 1 \div 1.43 = \% \text{ Fe} = 70\%.$$

$$43 \div 1.43 = \% \text{ O} = 30\%.$$

$$(b) \frac{70}{16} = 1.25. \text{ Divided by } .62 = \text{Fe}_2, \quad \}$$

$$\frac{30}{16} = 1.88. \text{ Divided by } .62 = \text{O}_3, \quad \}$$

$$(c) 1 : 43 :: x : 16$$

$$x = 37 = \text{eq. wt.}$$

VIRGINIA GREATEST SOAPSTONE STATE.

In the production of soapstone, the United States ranks first among all countries, and Virginia produces about twenty times as much as the four other producing states—Maryland, North Carolina, Rhode Island, and Vermont. The waste from breakage in quarrying, sawing into slabs, manufacturing, and final transportation is so great as to render success in the industry a matter of skillful manipulation. The value of the stone is in large measure proportionate to the work done upon it. In the rough it is valued at \$2 or less a ton, but when sawed into slabs its value is increased to about \$15, and when made into laundry tubs it may attain a value of about \$30 a ton. The production of soapstone and talc in the United States is steadily increasing, according to the United States Geological Survey, Department of the Interior. In 1900 it was 27,943 short tons, in 1910 it was 150,716 tons, and in 1915 it was 186,891 short tons.

UNCLE SAM'S MAPS AT A PREMIUM.

That Uncle Sam's topographic maps are appreciated by public utility corporations is shown by the fact that telephone companies throughout the United States are constant purchasers. These companies send frequent orders to the Geological Survey, Department of the Interior, for its maps in lots of 250 or 500, and occasionally when some big contract has been landed as many as 1,000 maps are ordered at a time for the use of the engineers and linemen. Some electrical supply companies keep complete sets of the maps of areas in states in which they expect to do extended work, and when they hear that contracts are to be let for such work they refer to these maps, and with the "lay of the land" before them can tell at a glance the character of the work that will be required and can make their bids promptly and intelligently. The telephone officials who are "using the maps extensively" state to the Survey that they are of "indispensable value" in their work.

IOWA ASSOCIATION OF MATHEMATICS TEACHERS.

The eighth annual meeting of the Iowa Association of Mathematics Teachers was held at Des Moines, November 5, 1916, in connection with the annual meeting of the State Teachers' Association.

The Association was called to order by the President, Miss Maud St. John, East Des Moines High School.

The minutes of the previous meeting and the financial report for 1915-16 were read and adopted.

The program was carried out as printed, and was as follows:

"Are There Dangers in the Modern Tendencies?"—Miss Wenonah Macey, West Des Moines High School.

"History and Use of Mathematical Texts"—Prof. G. A. Miller, University of Illinois, Urbana, Ill.

"Report of the Committee on Elimination of Unnecessary Material in Mathematics in Iowa"—Submitted by the Chairman, F. K. Williamson, Ottumwa High School.

"Elimination from the College Teacher's Standpoint" was presented as a special feature of the elimination report, by Prof. E. E. Watson, Parsons College, a member of the Elimination Committee.

All of the reports were adopted, placed on file, and the committees commended for their successful efforts.

The Committee on Nominations, appointed by the President—W. E. Beck, Miss Floe Correll, and W. A. Crusinberry—reported as follows on the officers for 1916-17:

President—F. K. Williamson, Ottumwa High School.

Vice-President—E. E. Watson, Parsons College, Fairfield.

Secretary-Treasurer—Ira S. Condit, Iowa State Teachers College, Cedar Falls.

The Secretary was instructed to cast the unanimous ballot of the Association for the officers as nominated.

On motion, the incoming President was requested to appoint a Committee on an Iowa Syllabus of Secondary Mathematics.

On motion, the Publicity Committee was allowed the sum of ten dollars for the next year's work.

The Secretary informed the Association of the serious illness of Prof. Arthur G. Smith, Head of the Department of Mathematics, State University of Iowa, an active and efficient member of the Association since its organization. By a rising vote, Mr. W. E. Beck, a member of Professor Smith's department, was requested to convey an expression of appreciation of helpful cooperation, and of sympathy in the hour of trial, to Professor and Mrs. Smith.

Adjournment.

The Iowa Association was organized in 1909 and took the place of the Mathematics Round Table which had existed for many years in connection with the State Teachers' Association. Its definite aim has been the improvement of the teaching of mathematics in Iowa. Each year a committee has been at work on some particular phase of the state work. Principal reports and addresses are as follows:

1909—Report on "Secondary Mathematics, Matter, and Method."

1910—Report on "Use of Graph in Elementary Algebra;" address on "Correlation of High School Mathematics," Miss Edith Long, Lincoln, Neb., High School.

1911—Report on "Abridging and Enriching the Course in Mathematics in Iowa;" address on "A Sensible Method of Teaching Arithmetic," Supt. G. M. Greenwood, Kansas City.

1912—Report on "Correlation of Secondary Mathematics;" report on "Elimination of Unnecessary Material from State Examination Questions in Mathematics;" address on "Changing Conditions as Related to the Mathematics of our Secondary Schools," Prof. H. E. Slaught, University of Chicago.

1914—Report on "Teaching Original Work in High School Mathematics;" address on "Recreations in Mathematics," Charles W. Newhall, Shattuck School, Faribault, Minn.

1915—Report of Publicity Committee (books and magazine articles); report on "Elimination of Unnecessary Material from Secondary Mathematics in Iowa;" address on "Efficiency in Mathematics Teaching with Special Reference to Standard Tests," Dr. S. A. Courtis, Detroit, Mich.

Reports are printed or mimeographed and placed in the hands of members for further study.

IRA S. CONDIT,
Secretary-Treasurer.

MINUTES OF THE INDIANA ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The twenty-second session of Indiana Association of Science and Mathematics Teachers was held at Indianapolis, October 26, 1916, and was called to order by President Montgomery in Caleb Mills Hall.

The reading of the minutes of the previous meeting was dispensed with. A letter from Mr. Tillman was read by Mr. Wade, requesting that a new Secretary-Treasurer be appointed if the meetings were to be held in the fall, as the former could not attend at that time. This raised the old question of time of meeting, and it was decided best to become a part of the State Teachers' Association, if the Executive Committee of that body were willing to provide for the section as they have the past two years.

The President then appointed the following committees:

Auditing Committee—Prof. Frank R. Higgins, State Normal; Mr. Orville Fidlar, Vincennes; Mr. Walter Brumfield.

Nominating Committee—Prof. F. T. Hodge, Franklin; Mr. G. W. Reed, Marion; Mr. A. H. Wagoner.

Dr. Aley then gave a splendid address on the subject, "Science and Progress," and was followed by Dr. Mees of Rose Polytechnic, who gave an illustrated lecture on "The Gyroscope—Some Practical Applications," which was entertaining and instructive.

The Auditing Committee reported that the Treasurer's statement was correct, and the report was accepted.

The Nominating Committee reported the following nominations:

President—Claude E. Kitch, E. M. T. H. S., Indianapolis, Ind.

Vice-President—Prof. Frank R. Higgins, State Normal, Terre Haute, Ind.

Members of Executive Committee—Prof. Edwin Morrison, Earlhorn; Prof. Virgil Smiley, Franklin.

Motion carried for the Secretary to cast the ballot for the entire list of officers.

Motion for the Executive Committee to fill the office of Secretary-Treasurer was carried.

The time for the next meeting was left in the hands of the Executive Committee, but preference was voted to have the meeting at the time of the State Teachers' Association.

The meeting adjourned.

CLAUDE E. KITCH,
Secretary pro tem.

THE PROPOSED NEW POSTAL LAW.

The house committee on postal service has unanimously recommended the adoption of the bill reducing postage on first class matter to one cent per ounce and increasing the rate on periodicals. The rate of postage on magazines will be governed by zones as is now done with parcel post matter. All magazines will be obliged to increase their subscription rates to meet this extra expense, in many cases over one dollar. With this Journal we will be obliged to raise the rate to \$2.50 per year in order to meet the burden of extra cost. The reading public will bear the extra load. We are asking all of our readers to write their congressman protesting against this unjust discrimination toward magazines, many of which will suspend operation if this proposed bill goes into operation. The journals thus put out of commission will be those of an educational character, and the very ones the educational world can least afford to lose. Write at once to your congressman, protesting against this discrimination.

CLASSROOM SAYINGS.

In a recent physics class which was studying the problems relating to lift and force pumps, a question was printed in the manual asking the pupils to describe the Holly waterworks system. A young man came to the instructor and asked where he could get some information on "the holy waterworks system."

A fuse is a small coil that is guaranteed to hold a certain amount of electricity.

A volt tells the amount of current, an ampere tells the strength of the current, an ohm tells both the amount and the strength of the current.

The advantage of a dry cell is to tell the amount of electricity, and the advantage of a gravity cell is to measure the amount of current.

The faster the armature, the more lines of force it collects.

Adenoids is a bunch that grows back of the nose. It keeps the nose from growing straight and it grows sidewise instead.

Antitoxin is what they give you before they operate.

Biceps muscle are around the abdomen; they are to expand the abdomen.

After one of our classes learned that at sea level the atmospheric pressure sustains a column of water about 34 feet high, a pupil in the class reasoned thus: "Mercury stands about 76 cm. high at sea level, because 62.4 lb. = weight of a cu. ft. of water, and 13.6 g. = weight of a cu. cm. of mercury, and adding the two we get 76 cm."

The following is a definition of "double decomposition" evolved by a girl pupil from an example submitted by the instructor:



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Bergen and Caldwell: Practical Botany - - - - - **\$1.30**
Introduction to Botany - - - - - **1.15**

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In conjunction with the Bergen Botanies, they are used in more schools than all other Botanies together!

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In the Central and Western States alone, more than 1500 schools have introduced the book in the nineteen months since its publication.

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The book that made General Science practicable for school use; that has gained great popularity in a very short time; that is causing a revision of the American high-school course.

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"Double decomposition is the giving off of some substance by another substance to be taken up by a third substance which gives up a fourth substance to be taken up by the first substance."

Can you follow her course of reasoning?

In his physiology, Johnny had read that one's intelligence did not necessarily depend upon the size of his brain, but upon the number and depth of its convolutions. As proof, an instance was cited of a comparison that had been made between the brain of a very intelligent woman and a crazy man. The man had the larger brain.

Johnny in recitation gave this startling bit of information: "Crazy people have large brains."

Biceps muscle are the means of people walking so far without getting tired.

Cilia is the sticky substance on the wing of a butterfly.

Cilia are the whiskers on an amoeba (amoeba?).

Cornea is the last bone in the spinal column.

Diagram (diaphragm) is the bony framework of the body.

Diaphragm is the part of your stomach you breath with. (The author of this was a student of elocution.)

Femur is the female organ of reproduction of a crayfish.

The girdle of an earthworm is its most sensitive part. Within it are its heart, lungs and respiratory organs. It purifies the blood and stores nutrition.

BOOKS RECEIVED.

Mortality Statistics, Fifteenth Annual Report, Bureau of the Census, Washington, D. C. 714 pages. 24x30 cm. Cloth. 1916. Government Printing Office, Washington, D. C.

Forty-Eighth Annual Insurance Report of the Insurance Superintendent of the State of Illinois, Rufus M. Potts, Superintendent. 851 pages. 16x22 cm. Cloth. 1916. Department of Insurance, Springfield, Ill.

Laboratory Guide to the Study of Qualitative Analysis, by E. H. S. Bailey and Hamilton P. Cady, University of Kansas. Pages x-294. 14.5x21 cm. Cloth. 1916. \$1.50 net. P. Blakiston's Son & Company, Philadelphia.

Field Management and Crop Rotation, by Edward C. Parker, formerly with the Minnesota Agricultural Experiment Station. 507 pages. 13.5x19.5 cm. Cloth. 1915. \$1.50.

Soils and Soil Fertility, by A. R. Witson and H. L. Walster, University of Wisconsin. 315 pages. 13.5x20 cm. Cloth. 1916. \$1.25 net.

Elements of Farm Practice, by C. D. Wilson, Chairman of Agricultural Extension, University of Minnesota, and E. W. Wilson. 347 pages. 13.5x20 cm. Cloth. 1916. \$1.00. The Webb Publishing Company, St. Paul, Minnesota.

How to Use Your Mind—A Psychology of Study, by Harry D. Kitson, University of Chicago. 215 pages. 12.5x18.5 cm. Cloth. 1916. \$1.00 net. J. P. Lippincott Company, Philadelphia.

Oral English or the Art of Speaking, by Antoinette Knowles, High School, San Jose, Cal. Pages vi-361. 13x19 cm. Cloth. \$1.20. D. C. Heath & Company, Boston.

Elementary Economic Geography, by Charles R. Dryer, formerly of the Indiana State Normal School. 415 pages. 14x20.5 cm. Cloth. 1916. American Book Company.

The Psychology of Drawing, by Fred C. Ayer, University of Oregon. Pages ix-186. 13.5x19.5 cm. Cloth. 1916. \$1.25. Warwick & York, Baltimore, Md.

Plane Geometry, by Edith Long, High School, Lincoln, Neb., and W. C. Brenke, University of Nebraska. Pages vii-576. 13x19 cm. Cloth. 1916. \$1.00. Century Company, New York City.

Five Hundred Practical Questions in Economics, by a Committee of the New England History Teachers' Association. 58 pages. 13.5x18.5 cm. Paper. 1916. 25 cents. D. C. Heath and Company, Boston.

Laboratory Manual of Chemistry in the Home, by Henry T. Weed, Manual Training High School, Brooklyn, New York. 200 pages. 19x24 cm. Loose leaf. 1916. American Book Company, New York City.

A VARIABLE SELF AND MUTUAL INDUCTOR.

The Bureau of Standards, Department of Commerce, has just published a paper which describes a new form of instrument for varying that property of an electrical circuit (self inductance) which opposes any change in the strength of a current, just as the inertia of a heavy train of cars opposes any change in its speed. It consists of two sets of coils of insulated wire mounted in circular hard rubber plates, between which a similar plate, carrying two coils, is arranged to turn, thus varying the inductance. Diagrams and data are given, from which instruments of this type may be designed to meet the requirements of a given use. Comparison is made of the new instrument and of older ones.

Pyrex Laboratory Glassware

Pyrex Glass—a new borosilicate glass possessing an extraordinarily low expansion coefficient, 0.0000032, and great resistance to sudden temperature changes.

Chemical stability tests show Pyrex glass to be less soluble in water and acids and about equally soluble in alkalis, compared with the best resistance glass, either American or foreign, hitherto offered. The glass contains no metals of the magnesia-lime-zinc group and no heavy metals.

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REPORT OF THE COMMITTEE ON VOCATIONAL MATHEMATICS.

Members of the Central Association of Science and Mathematics Teachers may obtain a complete report in mimeograph form on application to the chairman, Prof. K. A. Smith, Iowa State University, Ames, Iowa.

BOOK REVIEWS.

A Laboratory Guide to the Study of Qualitative Analysis, Based upon the Application of the Theory of Electrolytic Association and the Law of Mass Action, by E. H. S. Bailey, Ph. D., and Hamilton P. Cady, Ph. D., Professors of Chemistry in the University of Kansas. Eighth edition, revised by Paul V. Faragher, Ph. D., Assistant Professor of Chemistry in the University of Kansas in collaboration with the authors. Pages x+294. 15x21x1.7 cm. Charts of analytical separations. Cloth. 1916. \$1.50 net. P. Blakiston's Son & Company.

This revision modernizes the work, making it distinctly worth considering as a text, even in competition with some of the newer books. The order of treatment puts the alkali metals in the first group, and silver, lead, and mercury (mercurous) in Group VII; arsenic, antimony, tin, etc., being given a distinct grouping, rather than being put in a subdivision of the sulphide group. Anions are divided into five groups, and the consideration of them follows that of the metals.

The last eighty or ninety pages treat of methods of systematic analysis of unknown substances. F. B. W.

Laboratory Manual of Inorganic Chemistry for Colleges, by Lyman C. Newell, Ph. D. (Johns Hopkins), Professor of Chemistry, Boston University, Boston, Mass. Author of "Experimental Chemistry," "Descriptive Chemistry," "General Chemistry," and "Inorganic Chem-

istry for Colleges." Pages vi+240. 13x19x2 cm. Diagrams of apparatus. Cloth. 1916. D. C. Heath & Company.

More than 350 exercises for the laboratory are provided in this manual, giving the college instructor ample material so that he can select his own course according to his own ideas.

The directions are brief, but sufficient for the mature student. The order of topics is systematic. A well-arranged appendix contains a section on general laboratory directions and a very detailed list of laboratory equipment. The later experiments deal with the metals, and especially with their analytical reactions. While containing many experiments which have become the common property of chemical instruction, there are also numerous new experiments in the manual. F. B. W.

SOME AGRICULTURAL TEXTBOOKS FOR SECONDARY AND OTHER SCHOOLS.

1. *Soils and Soil Fertility*, by A. R. Whitson and H. L. Walster, University of Wisconsin. 315 pages. Illustrated. Cloth. \$1.25 net. Published by the Webb Publishing Company, St. Paul, Minn.
2. *Elements of Farm Practice*, by A. D. Wilson, University of Minnesota, and E. W. Wilson. 332 pages. Illustrated. Cloth. \$1.00. Published by the Webb Publishing Company, St. Paul, Minn.
3. *Field Management and Crop Rotation*, by Edward C. Parker, Special Agent, U. S. Department of Agriculture. 512 pages. Illustrated. Cloth. \$1.50. Published by the Webb Publishing Company, St. Paul, Minn.
4. *Field Crop Production*, by George Livingston, Ohio State University, 424 pages. Illustrated. Cloth. \$1.40. Rural Science Series of Textbooks, published by The Macmillan Company, New York.
5. *Principles of Agronomy*, by Franklin S. Harris and George Stewart, Utah Agricultural College. 451 pages. Illustrated. Cloth. \$1.40. Rural Science Series of Textbooks, published by The Macmillan Company, New York.
6. *Animal Husbandry for Schools*, by Merritt W. Harper, Cornell University. 400 pages. Illustrated. Cloth. \$1.40. Rural Science Series of Textbooks, published by The Macmillan Company, New York.
7. *Selected Readings in Rural Economics*. Compiled by Thomas Nixon Carver, Harvard University. 974 pages. Cloth. \$2.80. Published by Ginn & Company, Boston.

High school teachers of botany may enrich their courses in botany by judicious selections from the field of agriculture, and at the same time make their work immensely more interesting to their pupils. There is plenty of justification for such a course, for agriculture is an applied science based upon botany and other sciences. We may call this the botany of agriculture. There have recently come into the hands of the reviewer several textbooks which we shall review together, and from which teachers of botany may make good selections. It goes without saying that their primary purpose as textbooks of agriculture will not be lost sight of.

Soils and Soil Fertility. While attending the Wisconsin State Teachers' convention at Milwaukee not long ago, our attention was arrested by some textbooks displayed by the Webb Publishing Company that seemed particularly fresh and well organized. Knowing the sort of work being turned out by the State Universities of Wisconsin and Minnesota, we asked permission to take some of the books for review. We class the book on *Soils and Soil Fertility* as one of the best textbooks it has been our fortune to meet. It is well organized, its statements are clean-cut

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and vigorous, and there is no padding with useless words that get nowhere. The sections are titled, short, and always to the point. The book is teachable. So many authors are vague in their statements and use too many general statements that may mean anything. Such books are an abomination in the classroom. It is refreshing to get hold of a textbook that high school pupils can read and understand.

It must be mentioned also that there is a good manual with a series of well-chosen laboratory exercises on the soils. This manual may be of great use to the teacher of botany as a source of suitable exercises for his classes, as well as for the teacher of agriculture.

Elements of Farm Practice. While this book is similar in manner of presentation to *Soils and Soil Fertility*, it is written for younger pupils—for the rural schools and earlier years of the high schools. There is in places too great an effort to write down to the level of such pupils with language that must sound silly even to pupils of this age, and there is also some padding with matter that no country boy of "rural schools" needs, as, for example, instructions for shocking wheat. The space used for such instructions might better be used for a more adequate treatment of the soil. Knowledge of the soil is so fundamental to successful agriculture that it seems strange that textbook writers of agricultural textbooks so often give it such meager treatment.

However, aside from these defects, the book is well organized, and covers the field very adequately. The sections are short, and the treatment direct and well chosen. The illustrations are particularly well chosen and helpful. We imagine that some directions for simple exercises would be a very great help to the teachers for whom the book is intended. There are some good arithmetical problems, based on the text, at the close of chapters and many sections, but no experimental work is suggested. Many simple experiments could be carried on, even with the limited facilities of rural schools, which would add much to the worth-while-ness of the work. All in all, we think this book a very usable one.

Farm Management and Crop Rotation. This book is adapted to older students who have had training in the elementary sciences. It is written more in the style of the treatise for mature minds. The question of crop rotation is an important one, and such books as the one under discussion should be on the shelves of schools as a reference book, where not needed for a text.

The discussion is under six heads: "I. Historical Review;" "II. Rotations and Plans;" "III. Rotation and Commercial Fertilizers;" "IV. Experimental Evidence;" "V. Review of Soil Productivity;" "VI. Additional Features," including questions of soil inoculation, fungus diseases, etc. It will be seen that the treatment is comprehensive and practical. We can heartily commend this work.

Field Crop Production. This book covers the entire field of crop production. After two introductory chapters, there follow nineteen chapters on field crops, taking them up one after another in succession. Each crop therefore cannot be very fully discussed, but usually a historical sketch is given, followed by a description of botanical characters. Then the various types and varieties are described, methods of culture, etc.—finally concluding with an account of the plant's enemies. The book is written for agricultural colleges and schools, but will be useful for a reference book in other schools. The style and method of the book are good.

Principles of Agronomy. The authors state in their preface that this book is designed for schools having more than one course in agriculture. They also say that preceding courses in botany and chemistry, "although not presupposed," are desirable for a better understanding of the subject. On reading the book, we find that the authors are by no means consistent in thinking that a knowledge of botany is not presupposed. For example, there is no preliminary chapter on plant diseases, yet these diseases are discussed with the various field crops as if the student knew the fundamental facts. Just about one-half of the book is given to discussions of field crops. In our opinion, the discussion of the principles of agronomy should have a much larger share of any work on agronomy. We find what we might expect with such an overbalanced treatment, that some important topics are either omitted entirely, except by incidental mention, or slighted. Plant breeding is compressed into a chapter of twelve pages. Harmful insects and plant diseases have no separate treatment, and very scant mention anywhere. The discussion of soils is very good and comprehensive.

Probably the lack of balance is due largely to doubt as to just what is the function of a book on agronomy. There are very few agronomies written. It would seem to the reviewer that extended treatment of field crop production ought to be left to works making this a specialty. The agronomy may profitably include discussions of corn and wheat for many reasons, but these two cover all the principles of agronomy needed for an elementary work.

However, the book has its good points and will doubtless meet a need for a textbook of agronomy in schools of agriculture which give several courses in agriculture. High schools need a different sort of book, covering gardening, orcharding, plant breeding, plant enemies and their control, rotations and the legumes, as well as foundation work on soils, fertilizers, and tillage.



SCIENCE TEACHERS

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Animal Husbandry for Schools. This book was reviewed about three years ago. As the present text is merely a new edition and there has been no revision, it will not be necessary to make an extended review at this time. We heartily commend it to any schools wishing such a text. It is well written and well printed. It should be in high school libraries.

Selected Readings in Rural Economics. This is a good example of bookmaking. The thin paper makes it a handy volume in spite of the more than nine hundred pages. The type is clear and easy to read. The book is designed for agricultural schools, to be used as a handbook or reference book in courses on rural economics. There is considerable space given to the historical background as, for example, the rise of the Granger and Populist movements in this country. The selections have a wide range, including all phases of agricultural economics. We consider the book an important addition to agricultural literature, bringing together, as it does, for convenient use much scattered but highly important information. The publishers are to be commended, as well as Professor Carver, for the service they have rendered to agricultural education.

W. W.

The Morrison Outline Maps, by S. E. Morrison, Public School 132, Manhattan. 24 maps. 23.5x27.5 cm. and 27.5x31.3 cm. Pads of 50. 1916. 45 cents. Hinds, Hayden & Eldredge, New York City.

The list includes all the continents, British Isles, United States, Canada, Mexico, Central America, West Indies, five group maps of states arranged according to the plan followed in the report of the United States Commissioner of Education, Massachusetts, Connecticut, and Rhode Island, New York State, New Jersey, Pennsylvania, Illinois, City of New York, and City of Chicago. At first glance, they appear to be planned especially for the teaching of *commercial geography*, but one finds that they are as well suited to the teaching of *place geography, history, and civics*, while the suggestive lists in the margin make them attractive, also,

to the teacher who would emphasize the *economic* side of geology or geography. These marginal suggestions contain such lists as the following, though varying with the nature of the map: Rivers, Peninsulas, Islands, Lakes, Seas, Gulf, Bays, Mountains, Seaports, Cities, Productive Areas, Manufacturing Regions, Animals, Means of Communication, and subjects for special exercises.

The maps are perhaps too large to be conveniently placed in notebook covers of standard size (19.8x24.9 cm.), yet a reduction in size would be detrimental to their effective use. On the whole, they should receive a hearty welcome by progressive and busy teachers. C. M. W.

The Life of Inland Waters, by James G. Needham, Professor of Limnology in Cornell University, and J. T. Lloyd, Instructor in Limnology in Cornell University. 15x23 cm. with 438 pages and 244 illustrations in the text. 1916. Comstock Publishing Company, Ithaca, N. Y.

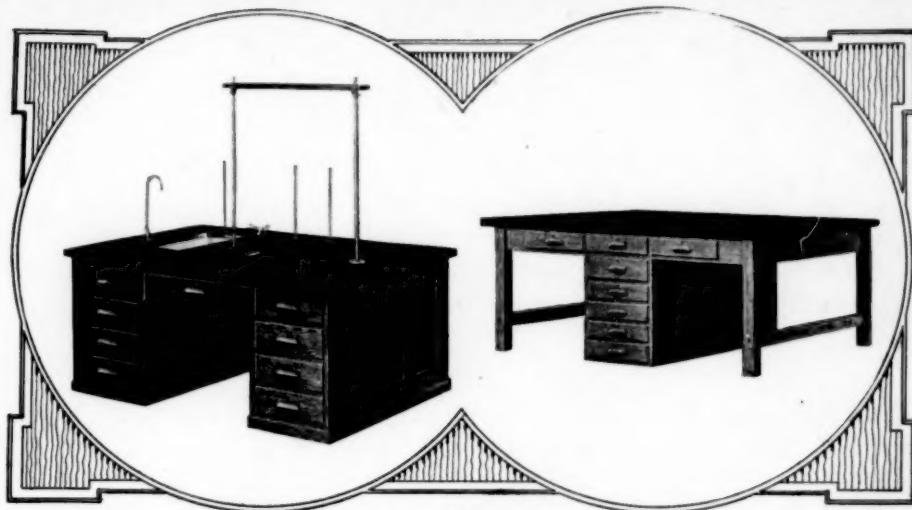
As stated on the title page, this is "an elementary textbook of fresh-water biology." The authors "endeavor to present a brief and untechnical account of fresh-water life, its forms, its conditions, its fitnesses, its associations, and its economic possibilities." To borrow again from the preface, our interests in water life are manifold. They may be enumerated as aesthetic, educational, sanitary, social, and civic interests. There are very few books of the scope of this one, taking in as it does all water life. The fact that it is written by Professor Needham and under his directions, and is a by-product of his work in Cornell, insures the high character and quality of the work.

The presentation of the subject matter is simple and untechnical, so far as possible in such an undertaking. The illustrations are profuse and original, many of them from photographs. Perhaps we can give no better idea of the nature of the contents of the volume than by giving the chapter heads as follows: "I. Introduction;" "II. The Nature of Aquatic Environment;" "III. Types of Aquatic Environment;" "IV. Aquatic Organisms;" "V. Adjustment to Conditions of Aquatic Life;" "VI. Aquatic Societies;" and "VII. Inland Water Culture," including "Aboriginal Water Culture, Water Crops and Water Culture and Civic Improvement."

For the high school biology teacher, there is a vast fund of information, and, best of all, it may be made a source book of stimulating ideas to arouse the interest of pupils. There is nothing so stimulating and so efficient for sustaining interest as the discovery of reasons for what the pupils observe, the discovery of causes and effects. We cannot speak too highly of its value for suggesting ways and means to the teacher—though, of course, the authors had in mind a college clientele in making their plans. W. W.

Differential and Integral Calculus, by Clyde E. Love, Ph., D., Assistant Professor of Mathematics in the University of Michigan. Pages vii+243. 14x21cm. 1916. The Macmillan Company, N. Y.

For most students, the value of a course in calculus lies chiefly in the ability gained to apply the principles in their later work. This book does not unduly stress the formal theoretical side of the subject, but gives precise statements of the fundamental principles involved, with explanations and illustrations which insure a clear understanding on the part of the student. There is little reason or occasion for a blind mechanical application of rules and formulas with this presentation of the subject. The development of methods of differentiating and integrating certain kinds of functions is followed by exercises and applications in sufficient



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number to fix in mind the process and its meaning. Moreover, the applications are selected with special reference to the needs of students of the engineering and mathematical sciences, and lay the foundation for intelligent work in these fields.

H. E. C.

Plane and Solid Geometry, by William Betz, Vice-Principal and Head of the Department of Mathematics in the East High School, Rochester, N. Y., and Harrison E. Webb, Head of the Department of Mathematics in the Central Commercial and Manual Training High School, Newark, N. J., with the editorial cooperation of Percy F. Smith, Sheffield Scientific School of Yale University. Pages xi+507. 13x19 cm. \$1.36. 1916. Ginn & Company, Boston.

The two parts of this book have been reviewed in this Journal. Teachers who desire a presentation of the subject midway between what may be called the extremely conservative and the extremely radical points of view will find this book of interest. In the omission of nonessential material, the logical and well-balanced sequence of the propositions retained, and the introduction of applied problems, graphs, and practical exercises, the long experience of the authors assures a teachable and serviceable book. The type page is clear, open, and attractive, and the diagrams and illustrations are admirable.

H. E. C.

Plane Geometry, by Fletcher Durrell, Head of the Mathematical Department, The Lawrenceville School, and E. E. Arnold, Specialist in Mathematics, The University of the State of New York. Pages 300. 13x19 cm. 88 cents. 1916. Charles E. Merrill Company, New York.

The list of propositions corresponding closely to the Harvard Syllabus and to the Report of the Committee of Fifteen, reduces by about one-fourth the number of propositions ordinarily included in a course in plane geometry. Facts known to the pupil are used as a natural approach to the subject, and construction work leads to the use of ruler, compasses, and protractor at once, and familiarizes the pupil with their use in working out exercises. The arrangement of proofs as numbered steps and reasons in parallel columns is helpful, both to pupil and teacher. The theory of limits and incommensurable cases are omitted. Improved methods of analysis and the group method of solving original exercises aid greatly in this part of the work. Attention is called frequently to the efficiency value of theorems and principles.

H. E. C.

Arithmetic for Engineers, by Charles B. Clapham, Lecturer in Engineering and Elementary Mathematics at the University of London, Goldsmiths' College, London. Pages xi+436. 15x22 cm. 5s. 6d. net, 1916. Chapman & Hall, Ltd., London.

It is no doubt true that most books of practical mathematics give so little attention to the explanation of the elementary mathematical processes that it is almost impossible for a man studying by himself to get a clear understanding of the methods and reasons in working out problems. This book presents no such difficulty, and it can be recommended for private study and for use in the classroom.

Every principle is explained in detail, and illustrated by worked examples. Since all the problems are those actually met with in the drafting room, shop, and laboratory, with data of correct dimensions, it rightfully is a part of the *Directly-Useful Technical Series*. The contents include not only arithmetic, but the essentials of algebra, mensuration, logarithms, graphs, and the use of the slide rule. A somewhat cursory examination reveals nothing in explanation or problem material that is not in accord with practice in this country, and in this sense it does not seem like a foreign book.

H. E. C.